Pulsars

http://www.cv.nrao.edu/course/astr534/Pulsars.html
http://www.nikhef.nl/pub/onderzoekschool/topics/Kuijpers1.pdf

High Energy Astrophysics

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Enrico Massaro notes
• Pulsed emission;
• Rotation and energetics;
• Magnetic field;
• Neutron star structure;
• Magnetosphere and pulsar models;
• Radiation mechanisms;
• Age and population
Introduction

• Known radio pulsars appear to emit short pulses of radio radiation with pulse periods between 1.4 ms and 8.5 seconds (P usually ≤ 1sec, dP/dt > 0). Even though the word *pulsar* is a combination of "pulse" and "star," pulsars are not pulsating stars. Their radio emission is actually continuous but beamed, so any one observer sees a pulse of radiation each time the beam sweeps across his line-of-sight. Since the pulse periods equal the rotation periods of spinning neutron stars, they are quite stable.
Introduction

• Radio observations of pulsars have yielded a number of important results because:

• **Neutron Stars** – supported by degeneracy pressure; Fermi *exclusion principle* restricts position hence Heisenberg *uncertainty principle* allows large momentum/high pressure - are physics laboratories providing extreme conditions (deep gravitational potentials, densities exceeding nuclear densities, magnetic field strengths as high as \( B \sim 10^{14-15} \) gauss) not available on Earth.

• **Pulse periods** can be measured with accuracies approaching 1 part in \( 10^{16} \), permitting exquisitely sensitive measurements of small quantities such as the power of gravitational radiation emitted by a binary pulsar system or the gravitational perturbations from planetary-mass objects orbiting a pulsar. Period increases quasi-regularly \( 10^{-20} < \frac{dP}{dt} < 10^{-12} \) s/s. However in some cases “glitches” are observed (abrupt decreasing of \( \frac{dP}{dt} \)).
Discovery

- The radical proposal that neutron stars exist was made with trepidation by Baade & Zwicky in 1934: "With all reserve we advance the view that a supernova represents the transition of an ordinary star into a new form of star, the **neutron star**, which would be the end point of stellar evolution. Such a star may possess a very small radius and an extremely high density." Pulsars provided the first evidence that neutron stars really do exist. They tell us about the strong nuclear force and the nuclear equation of state in new ranges of pressure and density, test general relativity and alternative theories of gravitation in both shallow and relativistically deep \((GM/rc^2)>>0\) potentials, and led to the discovery of the first extrasolar planets.

- Pulsars were discovered serendipidously in 1967 on chart-recorder records obtained during a low-frequency (\(\nu=81\) MHz) survey of extragalactic radio sources that scintillate in the interplanetary plasma, just as stars twinkle in the Earth's atmosphere.
Discovery

• Pulsar signals "had been recorded but not recognized" several years earlier with the 250-foot Jodrell Bank telescope. Most pulses seen by radio astronomers are just artificial interference from radar, electric cattle fences, etc., and short pulses from sources at astronomical distances imply unexpectedly high brightness temperatures $T \sim 10^{23} - 10^{30}\text{ K} \gg 10^{12}\text{ K}$, the upper limit for incoherent electron-synchrotron radiation set by inverse-Compton scattering.

• brightness temperature: $T_b = Fc^2 / k\nu^2$, the temperature for which if $F$ is given by the RJ formula ($F \sim KT\nu^2$) $T = T_b$.

• Cambridge University graduate student Jocelyn Bell noticed pulsars in her scintillation survey data because the pulses appeared earlier by about 4 minutes every solar day, so they appeared exactly once per sidereal day and thus came from outside the solar system.

• The sources and emission mechanism were originally unknown, and even intelligent transmissions by LGM ("little green men") were seriously suggested as explanations for pulsars.
Astronomers were used to slowly varying or pulsating emission from stars, but the natural period of a radially pulsating star depends on its mean density and is typically days, not seconds.
The Nobel Prize in Physics 1974

"for their pioneering research in radio astrophysics: Ryle for his observations and inventions, in particular of the aperture synthesis technique, and Hewish for his decisive role in the discovery of pulsars"

Sir Martin Ryle
United Kingdom
University of Cambridge
Cambridge, United Kingdom

Antony Hewish
United Kingdom

The Nobel Prize in Physics 1993

The discovery of the binary pulsar

During 1974 Joseph Taylor and Russell Hulse were searching for new pulsars with the Arecibo telescope. They discovered 40, one of which was to be very important.

When Hulse was observing the new pulsar, which has been named PSR1913+16, he found that the pulses arrived sometimes more often and sometimes less. The simplest interpretation was that the pulsar was orbiting another star very closely and at high velocity: Here one “pulsar year” is only about eight hours.

By observing the shift in the pulses, Hulse and Taylor found that the stars were equally heavy, each weighing about 1.4 times as much as the Sun. Since they were not visible on any photographs either, it was concluded that the other body, somewhat unexpectedly, was also a neutron star. Seen from Earth, however, it does not show up as a pulsar.
Basic properties

There is a lower limit to the rotation period $P$ of a gravitationally bound star, set by the requirement that the centrifugal acceleration at its equator not exceed the gravitational acceleration. If a star of mass $M$ and radius $R$ rotates with angular velocity $\Omega = \frac{2\pi}{P}$

\[
\Omega^2 R < \frac{GM}{R^2}
\]

\[
4\pi^2 R^3 \frac{1}{P^2} < GM
\]

\[
P^2 > \left( \frac{4\pi R^3}{3} \right) \frac{3\pi}{GM}
\]

\[
\rho > \frac{3\pi}{GP^2}
\]

\[
\rho = M \left( \frac{4\pi R^3}{3} \right)^{-1}
\]
Basic properties

Example: The first pulsar discovered (CP 1919+21, where the "CP" stands for Cambridge pulsar) has a period \( P = 1.3 \) s. What is its minimum mean density?

\[
\rho > \frac{3\pi}{GP^2} = \frac{3\pi}{6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ gm}^{-2}(1.3 \text{ s})^2} \approx 10^8 \text{ g cm}^{-3}
\]

This density limit is just consistent with the densities of white-dwarf stars. But soon the faster \((P = 0.033 \text{ s})\) pulsar in the Crab Nebula was discovered, and its period implied a density too high for any stable white dwarf. The Crab nebula is the remnant of a known supernova recorded by ancient Chinese astronomers as a "guest star" in 1054 AD, so the discovery of this pulsar also confirmed the Baade & Zwicky suggestion that neutron stars are the compact remnants of supernovae. The fastest known pulsar has \( P = 1.4 \times 10^{-3} \) s implying \( \rho > 10^{14} \text{ g cm}^{-3} \), the density of nuclear matter. For a star of mass greater than the Chandrasekhar mass

\[
M_{\text{Ch}} \approx \left( \frac{hc}{2\pi G} \right)^{3/2} \frac{1}{m_p^2} \approx 1.4M_\odot
\]
Basic properties

(compact stars less massive than this are stable as white dwarfs), the maximum radius is

$$R < \left( \frac{3M}{4\pi \rho} \right)^{1/3}$$

In the case of the \( P = 1.4 \times 10^{-3} \) pulsar with \( \rho > 10^{14} \text{ g cm}^{-3} \),

$$R < \left( \frac{3 \times 1.4 \times 2.0 \times 10^{33} \text{ g}}{4\pi \times 10^{14} \text{ g cm}^{-3}} \right)^{1/3} \approx 2 \times 10^6 \text{ cm} \approx 20 \text{ km}$$

The **canonical neutron star** has \( M \sim 1.4 \text{MSun} \) and \( R \sim 10 \text{ km} \), depending on the equation-of-state of extremely dense matter composed of neutrons, quarks, etc. The extreme density and pressure turns most of the star into a neutron superfluid that is a superconductor up to temperatures \( T \sim 10^9 \text{ K} \). Any star of significantly higher mass (\( M \sim 3 \text{MSun} \) in standard models) must collapse and become a black hole. The masses of several neutron stars have been measured with varying degrees of accuracy, and all turn out to be very close to 1.4MSun.
White dwarf, neutron stars and black holes

From the Salpeter IMF (number of stars formed each year per cubic Mpc with mass between $M$ and $M+dM$):

$$\psi(M, M+dM) = 2 \times 10^{-12} \ M^{-2.35} \ dm$$

98% of the stars will end forming a white dwarf, i.e. the total number of WD in a typical galaxy is $\sim 10^{10}$

Neutron stars will form if $M_{ns} > 1.4 M_{\text{Sun}}$. They are thought to form from the collapse of the core of stars with mass between 8 and 35 $M_{\text{Sun}}$. The total number of neutron stars in a typical galaxy is $\sim 10^9$.

Black holes will form if $M_{BH} > 3 M_{\text{Sun}}$. They are thought to form from the collapse of the core of stars with $M > 35 M_{\text{Sun}}$. The total number of black holes in a typical galaxy is $\sim 10^6$-$10^7$
Basic properties

The Sun and many other stars are known to possess roughly dipolar magnetic fields. Stellar interiors are mostly ionized gas and hence good electrical conductors. Charged particles are constrained to move along magnetic field lines and, conversely, field lines are tied to the particle mass distribution. When the core of a star collapses from a size $10^{11}$ cm to $10^6$ cm, its magnetic flux is conserved and the initial magnetic field strength is multiplied by $10^{10}$, the factor by which the cross-sectional area $a$ falls. An initial magnetic field strength of $B \sim 100$ Gauss becomes $B \sim 10^{12}$ Gauss after collapse, so young neutron stars should have very strong dipolar fields.
Magnetic induction

Magnetic flux,

\[ \int B dS = \text{constant} \]

Radius collapses from \(7 \times 10^7\) m to \(10^4\) m

Surface change gives

\[ \frac{B_{ns}}{B_{Sun}} = \left(\frac{7 \times 10^8}{10^4}\right)^2 \approx 5 \times 10^9 \]
• The Sun has magnetic fields of several different spatial scales and strengths but its general polar field varies with solar cycle and is \( \approx 0.01 \) Tesla.

• Thus the field for the neutron star:

\[
B_{\text{ns}} \sim 5 \times 10^7 \text{ Tesla} = 5 \times 10^{11} \text{ Gauss}
\]

• If the main energy loss from rotation is through magnetic dipole radiation then:

\[
B \sim 3.3 \times 10^{15} (\dot{P} P) ^{1/2} \text{ Tesla}
\]

or \( \sim 10^6 \) to \( 10^9 \) Tesla for most pulsars
Pulsar energetics

The traditional magnetic dipole model of a pulsar (Pacini 1966)

**Light cylinder** (the cylinder centered on the pulsar and aligned with the rotation axis at whose radius the co-rotating speed equals the speed of light).

\[ R_L = \frac{c}{\Omega} = \frac{c}{2\pi}P = 4.775 \times 10^9 \text{ cm} \]
Pulsar energetics

Larmor formula for the emission of a single accelerated charge $q$:

$$ P = \frac{2q^2\dot{v}^2}{3c^3} $$

Dipole approximation for many charges:
when there are many charges with positions $r_i$, velocities $v_i$ and charges $q_i, i = 1, 2 \ldots N$, we can find the radiation field at large distance by adding together the $E_i$ from each particle. However, the above expressions for the radiation fields refer to conditions at retarded times, which can be different for each particle. If the typical size of the system is $L$ and the typical time-scale over which significant changes of $E_{rad}$ occur is $\tau$, then the differences in retarded time across the source are negligible if $\tau >> L/c$. $\tau$ determines the characteristic frequency of the emitted radiation: $\nu \approx 1/\tau$.

Combining: $c/\nu >> L$, or $\lambda >> L$, i.e. the size of the source is smaller than the typical wavelength of the radiation. With the above condition we can write:

$$ P = \frac{2d^2}{3c^3} \quad \text{where} \quad d = \sum_i q_i r_i \quad \Rightarrow \quad \text{dipole moment, with the units of charge} \times \text{cm}.$$  

Analogously, one can define the magnetic dipole moment caused by a current loop carrying a current $I$ as

$$ m = \frac{1}{2c} I \int r \times ds = \frac{1}{c} IA \quad \text{again with units of charge} \times \text{cm} $$
Pulsar energetics

If the magnetic dipole is inclined by an angle $\alpha > 0$ from the rotation axis, it emits low-frequency radiation according to the following formula, analg of the Larmor formula:

$$P = \frac{2}{3} \frac{(\vec{m}_\perp)^2}{c^3}$$

where $m_\perp$ is the perpendicular component of the magnetic dipole moment.

$m$ is a function of $B$ and $r$:

$$\vec{B} = -\nabla \psi; \quad \text{where } \psi \text{ is the magnetic vector potential } \psi = \frac{\vec{m} \cdot \vec{r}}{r^3} = \frac{m \cos \theta}{r^2}$$

$$B_r = -\frac{\partial}{\partial r} \left( \frac{m \cos \theta}{r^2} \right) = \frac{2 m \cos \theta}{r^3}$$

$$B_\theta = -\frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{m \cos \theta}{r^2} \right) = \frac{m \cos \theta}{r^3}; \quad B_\varphi = 0$$

For a uniformly magnetized sphere with radius $R$ and surface magnetic field strength $B$:

$$m = BR^3$$
Pulsar energetics

If the inclined magnetic dipole rotates with angular velocity $\Omega$,

$$m = m_0 \exp(-i\Omega t)$$

$$\dot{m} = -i\Omega m_0 \exp(-i\Omega t)$$

$$\ddot{m} = \Omega^2 m_0 \exp(-i\Omega t) = \Omega^2 m$$

so

$$P_{\text{rad}} = \frac{2}{3} \frac{m_\perp^2 \Omega^4}{c^3} = \frac{2}{3} \frac{m_\perp^2}{c^3} \left(\frac{2\pi}{P}\right)^4 = \frac{2}{3} \frac{(BR^3 \sin \alpha)^2}{c^3} \left(\frac{2\pi}{P}\right)^4,$$

Where $P$ is the pulsar period. This electromagnetic radiation will appear at the very low frequency $\nu=1/P<1$kHz, so low that it cannot be observed, or even propagate through the ionized ISM. The huge power radiated is responsible for pulsar slowdown as it extracts rotational kinetic energy from the neutron star. The absorbed radiation can also light up a surrounding nebula, the Crab nebula for example.
Pulsar energetics

\[ P = \frac{2\pi}{\Omega}; \quad \frac{\partial P}{\partial t} = -\frac{2\pi}{\Omega^2} \frac{\partial \Omega}{\partial t}; \quad \frac{\partial^2 P}{\partial t^2} = \frac{2\pi}{\Omega^2} \left( \frac{2}{\Omega} \frac{\partial \Omega}{\partial t} - \frac{\partial^2 \Omega}{\partial t^2} \right) \]

\[ E_{\text{rot}} = \frac{1}{2} I \Omega^2 = \frac{2\pi^2 I}{P^2}; \quad I = \frac{2}{5} M R^2 \] for an homogeneous sphere

\[ M \sim 1 M_{\text{Sun}} = 2 \times 10^{33} \text{ g}; \quad R \sim 10\text{ km}; \quad I \sim 10^{45} \text{ g cm}^2, \] therefore the rotational energy of the Crab pulsar \((P = 0.033\text{s})\) is \(E_{\text{rot}} \sim 1.8 \times 10^{49} \text{ ergs}; \quad E_{\text{nucl}} = M c^2 \sim 2 \times 10^{54} \text{ ergs} \sim 10^5 E_{\text{rot}}\)

Pulsars are observed to slow down gradually:

\[ \dot{P} \equiv \frac{dP}{dt} > 0 \]

Note that \(\dot{P}\) is dimensionless (e.g., seconds per second). From the observed period \(P\) and period derivative \(\dot{P}\) we can estimate the rate at which the rotational energy is decreasing.

\[ \frac{dE_{\text{rot}}}{dt} = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = I \dot{\Omega} \dot{\Omega} \]
Pulsar energetics

\[ \Omega = \frac{2\pi}{P} \quad \text{so} \quad \dot{\Omega} = 2\pi(-P^{-2}\dot{P}) \]

\[ \frac{dE_{\text{rot}}}{dt} = I\dot{\Omega} = I\frac{2\pi^2}{P}(-\dot{P}) \]

Example: The Crab pulsar has \( P = 0.033 \) s and \( \dot{P} = 10^{-12.4} \). Rotational energy is changing at the rate

\[ \frac{dE_{\text{rot}}}{dt} = \frac{-4\pi^2 IP}{P^3} = \frac{-4\pi^2 \cdot 10^{45} \text{ g cm}^2 \cdot 10^{-12.4} \text{ s}^{-1}}{(0.033 \text{ s})^3} \approx -4 \times 10^{38} \text{ erg s}^{-1} \]

Thus the low-frequency (30 Hz) magnetic dipole radiation from the Crab pulsar radiates a huge power \( P_{\text{rad}} \approx -dE_{\text{rot}}/dt \approx 10^5 L_\odot \), comparable with the entire radio output of our Galaxy. It exceeds the Eddington limit, but that is not a problem because the energy source is not accretion. It greatly exceeds the average radio pulse luminosity of the Crab pulsar, \( \sim 10^{30} \text{ erg s}^{-1} \). The long-wavelength magnetic dipole radiation energy is absorbed by and powers the Crab nebula (a "megawave oven").
If we use $-\frac{dE_{\text{rot}}}{dt}$ to estimate $P_{\text{rad}}$, we can invert Larmor's formula for magnetic dipole radiation to find $B_{\perp} = B \sin \alpha$ and get a lower limit to the surface magnetic field strength $B > B \sin \alpha$, since we don't generally know the inclination angle $\alpha$.

$$P_{\text{rad}} = -\frac{dE_{\text{rot}}}{dt}$$
Pulsar energetics

\[
\frac{2}{3c^3} (BR^3 \sin \alpha)^2 \left( \frac{4\pi^2}{P^2} \right)^2 = \frac{4\pi^2 IP'}{P^3}
\]

\[
B^2 = \frac{3c^3 IP \dot{P}}{2 \cdot 4\pi^2 R^6 \sin^2 \alpha}
\]

\[
B > \left( \frac{3c^3 I}{8\pi^2 R^6} \right)^{1/2} (P \dot{P})^{1/2}
\]
Pulsar energetics

so the minimum magnetic field strength at the pulsar surface is

\[
\left( \frac{B}{\text{Gauss}} \right) > 3.2 \times 10^{19} \left( \frac{PP}{s} \right)^{1/2}
\] (6A4)

Example: What is the minimum magnetic field strength of the Crab pulsar \((P = 0.033 \, \text{s}, \, P' = 10^{-12.4})\)?

\[
\left( \frac{B}{\text{Gauss}} \right) > 3.2 \times 10^{19} \left( \frac{0.033 \, \text{s} \cdot 10^{-12.4}}{s} \right) = 4 \times 10^{12}
\]

\[
U_B = \frac{B^2}{8\pi} > 5 \times 10^{23} \, \text{erg cm}^{-3}
\]

Just one cm\(^3\) of this magnetic field contains over \(5 \times 10^{16} \, \text{J} = 5 \times 10^{16} \, \text{W s} = 1.6 \times 10^9 \, \text{W yr of energy, the annual output of a large nuclear power plant. A cubic meter contains more energy than has ever been generated by mankind.}\)
Emission mechanism

The neutron star is surrounded by a magnetosphere with free charges that produce intense electric currents. The neutron star is a spinning magnetic dipole, it acts as a unipolar generator. There are two main regions:

1. The closed magnetosphere, defined as the region containing the closed field lines within the light cylinder
2. The open magnetosphere, the region where the field lines cannot close before RL and extend above this radius.

If gravity is negligible, The total force acting on a charged particle is:

\[ \vec{F} = m \frac{dv}{dt} = q(\vec{E} + \vec{\beta} \times \vec{B}) \]

if \( m \to 0 \) \( \Rightarrow \) \( \vec{E} + \vec{\beta} \times \vec{B} = 0 \)

\( \vec{\beta} = \vec{\beta}_\parallel + \vec{\beta}_\perp \) with respect to the direction of \( B \) therefore:

\( \vec{E} = -\vec{\beta}_\perp \times \vec{B} \) and \( \vec{\beta}_\perp = \frac{\Omega \times \vec{r}}{c} \) corotation velocity.
Pulsar Magnetospheres

Forces exerted on particles

Particle distribution determined by

- gravity
- electromagnetism

Gravity

\[ F_{g_{ns}} = m_e g_{ns} = 9 \times 10^{-31} \times 10^{12} \approx 10^{-18} \text{ Newton} \]
Magnetic force

\[ F_B = evB = 1.6 \times 10^{-19} \times \frac{2\pi \left(10^4 m\right)}{33 \times 10^{-3} s} \left(10^8 T\right) \]

\[ \approx 3 \times 10^{-5} \text{ Newton} \]

This is a factor of \(10^{13}\) larger than the gravitational force and thus dominates the particle distribution.
Emission mechanism

The particles move along the field lines and at the same time rotate with them. Charges in the magnetic equatorial region redistribute themselves by moving along closed field lines until they build up an electrostatic field large enough to cancel the magnetic force and give \( F=0 \). The voltage induced is about \( 10^6 \) V in MKS units. However, the co-rotating field lines emerging from the polar caps cross the light cylinder (the cylinder centered on the pulsar and aligned with the rotation axis at whose radius the co-rotating speed equals the speed of light) and these field lines cannot close. Electrons in the polar cap are magnetically accelerated to very high energies along the open but curved field lines, where the acceleration resulting from the curvature causes them to emit curvature radiation that is strongly polarized in the plane of curvature. As the radio beam sweeps across the line-of-sight, the plane of polarization is observed to rotate by up to 180 degrees, a purely geometrical effect. High-energy photons produced by curvature radiation interact with the magnetic field and lower-energy photons to produce electron-positron pairs that radiate more high-energy photons. The final results of this cascade process are bunches of charged particles that emit at radio wavelengths.
Magnetosphere Charge Distribution

- Rotation and magnetic polar axes shown co-aligned
- Induced E field removes charge from the surface so charge and currents must exist above the surface – the Magnetosphere
- Light cylinder is at the radial distance at which rotational velocity of co-rotating particles equals velocity of light
- Open field lines pass through the light cylinder and particles stream out along them
- Feet of the critical field lines are at the same electric potential as the Interstellar Medium
- Critical field lines divide regions of + ve and – ve current flows from Neutron Star magnetosphere
A more realistic model...

- For pulses, magnetic and rotation axes cannot be co-aligned.
- Plasma distribution and magnetic field configuration complex for Neutron Star
- For $r < r_c$, a charge-separated co-rotating magnetosphere
- Particles move only along field lines; closed field region exists within field-lines that touch the velocity-of-light cylinder
- Particles on open field lines can flow out of the magnetosphere
- Radio emission confined to these open-field polar cap regions
A better picture

Neutron star
mass = 1.4 M☉
radius = 10 km
\( B = 10^4 \) to \( 10^9 \) Tesla
Curvature radiation

A charged particle forced to move along a curved trajectory with curvature $\xi$ will emit radiation. To find the power emitted we can make an analogy with the synchrotron emission, by introducing an "equivalent magnetic field" $B_{eq}$ intensity great enough to produce the observed curvature:

$$B_{eq} = \frac{pc}{e} \zeta = \frac{\gamma mc^2}{e} \beta \zeta$$

$$P_{CR} = \frac{2}{3} \frac{e^4}{m_e^2 c^3} \beta^2 \gamma^2 B_{eq}^2 = \frac{2}{3} ce^2 \gamma^4 \zeta^2$$  \quad \text{if } \beta \sim 1$$

$$\nu_{c} \approx \frac{\gamma^2 \omega}{\pi} = \frac{\gamma^3 \beta c}{\pi \rho}$$

$$(h\nu)_{CR} \approx \frac{ch\gamma^3}{\pi \rho} = \frac{ch\gamma^3}{\pi} \zeta$$  \quad \text{if } \rho \sim 10^6 \text{cm} \quad \text{and } \gamma \sim 10^6 \quad (h\nu)_{CR} \sim \text{GeV}$$

The extremely high brightness temperatures are explained by coherent radiation. The electrons do not radiate as independent charges $e$; instead bunches of $N$ electrons in volumes whose dimensions are less than a wavelength emit in phase as charges $Ne$. Since Larmor's formula indicates that the power radiated by a charge $q$ is proportional to $q^2$, the radiation intensity can be $N$ times brighter than incoherent radiation from the same total number $N$ of electrons. Because the coherent volume is smaller at shorter wavelengths, most pulsars have extremely steep radio spectra. Typical (negative) pulsar spectral indices are $\alpha \sim 1.7$ ($S \propto \nu^{-1.7}$), although some can be much steeper ($\alpha > 3$) and a handful are almost flat ($\alpha \sim 0.5$).
Radiation Mechanisms in Pulsars

Emission mechanisms

Total radiation intensity

- Summed intensity of spontaneous radiation of individual particles
  - exceeds coherent
  - does not exceed incoherent
Incoherent emission - example

For radiating particles in thermodynamic equilibrium i.e. thermal emission.

Blackbody $\Rightarrow$ max emissivity

So is pulsar emission thermal?
Consider radio: $\nu \sim 10^8$ Hz or 100MHz; $\lambda \sim 3m$
Use Rayleigh-Jeans approximation to find $T$: 

$$I(\nu) = \frac{2kT\nu^2}{c^2} \text{ Watts m}^{-2} \text{ Hz}^{-1} \text{ster}^{-1}$$

Crab flux density at Earth, $F \sim 10^{-25}$ watts m$^{-2}$ Hz$^{-1}$

Source radius, $R \sim 10\text{km}$ at distance $D \sim 1\text{kpc}$

then:

$$I(\nu) = \frac{F}{\Omega} = F\left(\frac{D^2}{R^2}\right) = \frac{10^{-25}\left(3 \times 10^{19}\right)^2}{\left(10^4\right)^2} \quad (1)$$
So -

\[ I_\nu = 10^6 \text{ watts m}^{-2}\text{Hz}^{-1}\text{ster}^{-1} \]

From equation (1):

\[ T = \frac{I(\nu)c^2}{2k\nu^2} K = \frac{10^6 \left(3 \times 10^8\right)^2}{2 \times 1.4 \times 10^{-23} \left(10^8\right)^2} K \]

\[ = 3 \times 10^{29} K \]

this is much higher than a radio blackbody temperature!
Models of Coherent Emission

high-$B$ sets up large pd $\Rightarrow$ high-E particles

electron-positron pair cascade

cascades results in bunches of particles which can radiate coherently in sheets
Emission processes in pulsars

- Important processes in magnetic fields:
  - cyclotron
  - synchrotron

- Curvature radiation

Optical & X-ray emission in pulsars

Radio emission

High magnetic fields; electrons follow field lines very closely, pitch angle $\sim 0^\circ$
Curvature vs Synchrotron

Synchrotron

Curvature
• Spectrum of curvature radiation (c.r.)
  - similar to synchrotron radiation,

\[
\text{Flux} \propto \nu^{1/3} \exp(-\nu)
\]

• For electrons:
  intensity from curvature radiation \(\ll\) cyclotron or synchrotron

• If radio emission produced this way, need coherence
Incoherent X-ray emission?

• In some pulsars, eg. Crab, there are also pulses at IR, optical, X-rays and γ-rays.

• - Are these also coherent?

• Probably not – brightness temperature of X-rays is about $10^{11}$ K, equivalent to electron energies 10MeV, so consistent with incoherent emission.
Beaming of pulsar radiation

• Beaming => radiation highly **directional**
• Take into account
  - radio **coherent**, X-rays and Optical **incoherent**
  - location of radiation source depends on frequency
  - radiation is directed along the magnetic field lines
  - pulses only observed when beam points at Earth

• **Model:**
  - radio emission from magnetic poles
  - X-ray and optical emission from light cylinder
Pulsar age

If \((B \sin \alpha)\) doesn't change significantly with time, we can estimate a pulsar's age \(\tau\) from \(PP'\) by assuming that the pulsar's initial period \(P_0\) was much shorter than the current period. Starting with

\[
B^2 = \frac{3c^3 IP \dot{P}}{8\pi^2 R^6 \sin^2 \alpha}
\]

calculated, we find that

\[
PP' = \frac{8\pi^2 R^6 (B \sin \alpha)^2}{3c^3 I}
\]

doesn't change with time. Rewriting the identity \(PP' = PP'\) as \(PdP = PP' dt\) and integrating over the pulsar's lifetime \(\tau\) gives

\[
\int_{P_0}^{P} PdP = \int_{0}^{\tau} (PP') dt = PP' \int_{0}^{\tau} dt
\]
Pulsar age

since \( P^2 \) is assumed to be constant over time.

\[
\frac{P^2 - P_0^2}{2} = PP' \tau
\]

If \( P_0^2 \ll P^2 \), the \textbf{characteristic age} of the pulsar is

\[
\tau \equiv \frac{P}{2P'} \tag{6A5}
\]

Note that the characteristic age is not affected by uncertainties in the radius \( R \), moment of inertia \( I \), or \( B \sin \alpha \); the only assumptions in its derivation are that \( P_0 \ll P \) and that \( PP' \) (i.e. \( B \)) is constant.

Example: What is the characteristic age of the Crab pulsar \( (P = 0.033 \text{ s}, \dot{P} = 10^{-12.4}) \)?

\[
\tau = \frac{P}{2\dot{P}} = \frac{0.033 \text{ s}}{2 \cdot 10^{-12.4}} \approx 4.1 \times 10^{10} \text{ s} \approx \frac{4.1 \times 10^{10} \text{ s}}{10^{7.5} \text{ s yr}^{-1}} \approx 1300 \text{ yr}
\]

Its actual age is about 950 years.
The *P-Pdot Diagram* is useful for following the lives of pulsars, playing a role similar to the Hertzsprung-Russell diagram for ordinary stars. It encodes a tremendous amount of information about the pulsar population and its properties, as determined and estimated from two of the primary observables, $P$ and $Pdot$. Using those parameters, one can estimate the pulsar age, magnetic field strength $B$, and spin-down power $dE/dt$. (From the Handbook of Pulsar Astronomy, by Lorimer and Kramer)
Pulsar population

Pulsars are born in supernovae and appear in the upper left corner of the PP diagram. If $B$ is conserved, they gradually move to the right and down, along lines of constant $B$ and crossing lines of constant characteristic age.

Pulsars with characteristic ages <$10^5$ yr are often found in SNRs. Older pulsars are not, either because their SNRs have faded to invisibility or because the supernova explosions expelled the pulsars with enough speed that they have since escaped from their parent SNRs. The bulk of the pulsar population is older than $10^5$ yr but much younger than the Galaxy ($10^{10}$ yr). The observed distribution of pulsars in the $PPdot$ diagram indicates that something changes as pulsars age. One possibility is that the magnetic fields of old pulsars decays on time scales $10^7$ yr, causing pulsars to move straight down in the diagram until they fall below into the graveyard below the death line.
Pulsar population

The death line in the $PPdot$ diagram corresponds to neutron stars with sufficiently low $B$ and high $P$ that the curvature radiation near the polar surface is no longer capable of generating particle cascades ($P_{\text{rad}}$ scales with $B^2$ and $P^{-4}$).

Almost all short-period pulsars below the spin-up line near $\log[Pdot/P (\text{sec})] \sim -16$ are in binary systems, as evidenced by periodic (i.e. orbital) variations in their observed pulse periods. These recycled pulsars have been spun up by accreting mass and angular momentum from their companions, to the point that they emit radio pulses despite their relatively low magnetic field strengths $B \ 10^8 \text{ G}$ (the accretion causes a substantial reduction in the magnetic field strength). The magnetic fields of neutron stars funnel ionized accreting material onto the magnetic polar caps, which become so hot that they emit X-rays. As the neutron stars rotate, the polar caps appear and disappear from view, causing periodic fluctuations in X-ray flux; many are detectable as X-ray pulsars.
Millisecond pulsars (MSPs) with low-mass ($M \sim 0.1-1 \, M_{\text{Sun}}$) white-dwarf companions typically have orbits with small eccentricities. Pulsars with extremely eccentric orbits usually have neutron-star companions, indicating that these companions also exploded as supernovae and nearly disrupted the binary system. Stellar interactions in globular clusters cause a much higher fraction of recycled pulsars per unit mass than in the Galactic disk. These interactions can result in very strange systems such as pulsar-main-sequence-star binaries and MSPs in highly eccentric orbits. In both cases, the original low-mass companion star that recycled the pulsar was ejected in an interaction and replaced by another star. (The eccentricity $e$ of an elliptical orbit is defined as the ratio of the separation of the foci to the length of the major axis. It ranges between $e$ for a circular orbit and $e$ for a parabolic orbit.) A few millisecond pulsars are isolated. They were probably recycled via the standard scenario in binary systems, but the energetic millisecond pulsars eventually ablated their companions away.
Neutron Stars

- **General parameters:**
  - \( R \sim 10 \text{ km } (10^4 \text{ m}) \)
  - \( \rho_{\text{inner}} \sim 10^{18} \text{ kg m}^{-3} = 10^{15} \text{ g cm}^{-3} \)
  - \( M \sim 1.4 - 3.2 \, \text{M}_\odot \)
  - surface gravity, \( g = GM/R^2 \sim 10^{12} \text{ m s}^{-2} \)

- **We are going to find magnetic induction, \( B \), for a neutron star.**
Neutron star structure

Neutron star segment

1. Solid core?
2. Superfluid neutrons, superconducting p+ and e-
3. Crystallization of neutron matter, $10^{18}$ kg m$^{-3}$

Heavy nuclei (Fe) find a minimum energy when arranged in a crystalline lattice

- $2 \times 10^{17}$ kg m$^{-3}$
- $4.3 \times 10^{14}$ kg m$^{-3}$
- $10^9$ kg m$^{-3}$
Regions of NS Interior

Main Components:

(1) Crystalline solid crust
(2) Neutron liquid interior
   - Boundary at $\rho = 2.10^{17}$ kg/m$^3$ – density of nuclear matter

Outer Crust:
- Solid; matter similar to that found in white dwarfs
- Heavy nuclei (mostly Fe) forming a Coulomb lattice embedded in a relativistic
degenerate gas of electrons.
- Lattice is minimum energy configuration for heavy nuclei.

Inner Crust (1):
- Lattice of neutron-rich nuclei (electrons penetrate nuclei to combine with protons and
  form neutrons) with free degenerate neutrons and degenerate relativistic electron gas.
- For $\rho > 4.3.10^{14}$ kg/m$^3$ – the neutron drip point, massive nuclei are unstable and
  release neutrons.
- Neutron fluid pressure increases with $\rho$
Regions of NS Interior (Cont.)

Neutron Fluid Interior (2):
- For $1 \text{ km} < r < 9 \text{ km}$, ‘neutron fluid’ – superfluid of neutrons and superconducting protons and electrons.
- Enables B field maintenance.
- Density is $2.10^{17} < \rho < 1.10^{18} \text{ kg/m}^3$.
- Near inner crust, some neutron fluid can penetrate into inner part of lattice and rotate at a different rate – glitches?

Core:
- Extends out to ~ 1 km and has a density of $1.10^{18} \text{ kg/m}^3$.
- Its substance is not well known.
- Could be a neutron solid, quark matter or neutrons squeezed to form a pion concentrate.
White Dwarfs and Neutron Stars

• In both cases, zero temperature energy – the Fermi energy, supports the star and prevents further collapse
• From exclusion principle, each allowed energy state can be occupied by no more than two particles of opposite spin
• Electrons in a White Dwarf occupy a small volume and have very well defined positions – hence from uncertainty principle, they have large momentum/energy and generate a high pressure or electron degeneracy pressure
• Corresponding “classical” thermal KE would have \( T \sim 3 \times 10^4 \) K and the related electron degeneracy pressure supports the star
• For a high mass stellar collapse, inert Fe core gives way to a Neutron Star and neutron degeneracy pressure supports the star
• NS has \( \sim 10^3 \) times smaller radius than WD so neutrons must occupy states of even higher Fermi energy (\( E \sim 1 \) MeV) and resulting degeneracy pressure supports NS
Low Mass X-ray Binary provides Observational Evidence of NS Structure

Neutron star primary

Accretion disk

Roche point

Evolved red dwarf secondary
Gravitationally Redshifted Neutron Star Absorption Lines

• XMM-Newton found red-shifted X-ray *absorption* features

  - observed 28 X-ray bursts from EXO 0748-676

• Fe XXVI & Fe XXV (n = 2 – 3) and O VIII (n = 1 – 2) transitions with z = 0.35

• Red plot shows:
  - source continuum
  - absorption features from circumstellar gas

• Note: \( z = \frac{\lambda - \lambda_o}{\lambda_o} \) and \( \frac{\lambda}{\lambda_o} = (1 - 2GM/c^2r)^{-1/2} \)
X-ray absorption lines

- quiescence
- low-ionization circumstellar absorber

- Low T bursts
  - Fe XXV & O VIII
  - (T < 1.2 keV)

- High T busts
  - Fe XXVI
  - (T > 1.2 keV)

redshifted, highly ionized gas

- $z = 0.35$ due to NS gravity suggests:
- $M = 1.4 - 1.8 \, M_\odot$
- $R = 9 - 12 \, \text{km}$
EXO0748-676

origin of X-ray bursts

circumstellar material
Observational Evidence for Pulsar Emission Sites

• Radio pulses come from particles streaming away from the NS in the magnetic polar regions:
  – Radio beam widths
  – Polarized radio emission
  – Intensity variability

• Optical and X-ray brightening occurs at the light cylinder
  – Radiation at higher energies only observed from young pulsars with short periods
  – Only eight pulsar-SNR associations from more than 500 known pulsars

• Optical and X-radiation source located inside the light cylinder
  – Pulse stability shows radiation comes from a region where emission position does not vary
  – High directionality suggests that emission is from a region where field lines are not dispersed in direction i.e. last closed field lines near light cylinder
  – Regions near cylinder have low particle density so particles are accelerated to high energies between collisions
In summary...

• **Radio emission**
  - coherent
  - curvature radiation at polar caps

• **X-ray emission**
  - incoherent
  - synchrotron radiation at light cylinder
Pulsar Population

• To sustain this population then, 1 pulsar must form every 50 years.
• cf SN rate of 1 every 50-100 years
• only 8 pulsars associated with visible SNRs (pulsar lifetime 1-10million years, SNRs 10-100 thousand... so consistent)

• but not all SN may produce pulsars!!!
Light Cylinder

- Radiation sources close to surface of light cylinder

- Simplified case – rotation and magnetic axes orthogonal