THE ASTROPHYSICAL JOURNAL, 376:L37–L40, 1991 August 1

THE MERGING RUNAWAY

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Received 1991 January 16; accepted 1991 May 2

ABSTRACT

We propose an analytic description of the direct mergings that involve galaxies in groups. We predict shape and evolution of the distribution of the galaxian masses in these finite systems, including the formation of a cD-like merger that analytically appears as a critical phenomenon. Our findings constitute a close counterpart of many N-body results. Similar aggregation phenomena conceivably take place in other cosmic structures.

Subject headings: cosmology — galaxies: clustering

1. THE PROBLEM

Many direct N-body experiments (Carnevali, Cavaliere, & Santangelo 1981; Ishizawa et al. 1983; Barnes 1989) show the occurrence of intense merging activity in simulated groups of galaxies.

In fact, these authors found that in such systems, slow hyperbolic encounters of galaxies may be so effectively inelastic due to energy transfer from orbital to internal degrees of freedom, as to cause prompt and multiple galaxy mergings, which drive drastic evolution of the whole system in some dynamical times.

Often these simulations end up in the formation of a large central object with a brightness profile similar to a cD galaxy, surrounded by a few, small orbiting galaxies in a shrunken configuration (Carnevali et al. 1981; Cavaliere et al. 1983; Barnes 1989). The authors stress the similarity of their results to observed groups dominated by a cD-like galaxy (Albert, White, & Morgan 1977; Tonry 1987).

In this *Letter*, we propose an analytical structure that captures an essential aspect of these phenomena and indicates interesting extensions.

2. THE FRAMEWORK

Effective descriptions and predictions of the evolving distribution $N(M, t)$ of galaxy masses call for a time-resolved kinetic equation expressing dN/dt. The right-hand side of such an equation (Cavaliere & Colafrancesco 1990; Cavaliere, Colafrancesco, & Scaramella 1991) will have a linear form $\propto N$ when evolution is driven mainly by direct collapses of primordial density perturbations in a hierarchical sequence (see Peebles 1980). It will have instead a nonlinear structure when the main role is played by interactions of collapsed units aggregating into higher hierarchical levels. Estimates starting from collapse theories (see White & Rees 1978) indicate that galaxies in groups are likely to interact with companion members over a few to several crossing times.

In the following we focus on such interactions and take up the formalism of the classic aggregation equation (Smoluchowski 1916). Its possible cosmogonic relevance has been pointed out repeatedly (see Silk & White 1978; Lucchin 1988), while its rich analytic content has been unveiled only recently (see Ernst 1986). We write

$$\frac{\partial N}{\partial t} = \frac{1}{2}\int_0^M dM' K(M', M - M', t) N(M', t) N(M - M', t) - N(M, t)\int_0^{M_{\max}} dM' K(M, M', t) N(M', t) , \quad (2.1)$$

for the distribution function $N(M, t)$ normalized to the system mass $\mathcal{M}$, taken in "comoving" form on dividing by ρ_a, the ambient mass density in the group. In the intrinsically *finite* systems we will consider, the lower integration limits actually mean the smallest mass in the system $M_{\min} \ll \mathcal{M}$ (in general, the equation may be formulated in an alternative form free of any apparent infinity). The upper limit is the largest mass $M_{\max} < \mathcal{M}$.

The interaction kernel $K = \rho_a \langle \Sigma V \rangle$ includes the velocity-averaged gravitational cross section for encounters of two galaxies with masses M and M' and relative velocity V. This reads $\Sigma \approx \pi(r + r')^2 v^2/V^2$ for focused, resonant interactions (FI); see Saslaw (1985). The symmetrized form $v^2 = 2G(M + M')/(r + r')$ applies in the interesting range of mass ratios $\sim 10^1$, and the condition $V/v < \alpha \sim$ a few is most effective for mergings. As for the galactic radii, we mainly consider the simple scaling $r \propto (M/\rho)^{1/3}$ to apply during the process, ρ being the internal density; an alternative scaling is discussed in § 6.

So $\langle \Sigma V \rangle$ is a homogeneous function of M, M', with degree $\lambda = 4/3$ for FI. In terms of a characteristic mass $M_*(t)$ and of the normalized mass $m \equiv M/M_*$, its scaling reads $\langle \Sigma V \rangle \propto M_*^{4/3} V^{-1} \rho^{-1/3} \psi(m, m')$, with $\psi(m, m') \propto (m^{1/3} + m'^{1/3})(m + m')$.

We show that for such $\lambda > 1$, equation (2.1) implies the occurrence of a critical phenomenon.

3. RUNAWAY IN GROUPS OF GALAXIES

First we argue heuristically, on the basis of the merging rate

$$\tau^{-1} \sim n\langle \Sigma V \rangle \propto \frac{\rho_a}{\rho^{1/3} V} M_*^{1/3} , \quad (3.1)$$

holding when the mass $\mathcal{M}$ in the set of normal galaxies is conserved, and their number density scales as $n \propto \rho_a/M_*$. A rate $\tau^{-1}(t)$ accelerating with time will point to the onset of a runaway process.

Because $M_*(t)$ will grow by mergings, a sufficient condition for FI to drive τ^{-1} to run away requires the time-depending coefficient $\mathscr{F}(t) \propto \rho_a^{2/3}(\rho_a/\rho)^{1/3} V^{-1}$ not to decrease rapidly; if so, a *positive* feedback loop for $M_*(t)$ may set in. In the open intergalactic "field," such a condition will be easily impaired by rarefaction of the ambient in cosmic time ($\rho_a \propto t^d$, with $d = -2$ to -3 when $\Omega_0 = 1$ to 0), and by the heating up of the linear velocity field ($V \propto t^{1/3}$; see Vittorio & Turner 1987). Within a group, instead, ρ_a is expected to rise during the virialization phase and to grow slowly thereafter, while the velocity dispersion V only undergoes a modest increase. Then $M_*(t)$ may start a runaway growth over some crossing times $t_d \sim R/V$ of the group, since $\tau/t_d \sim 1/n\Sigma R \lesssim 10^1$ holds.

In galaxy groups the velocity field is close to resonant conditions. With a mass ratio of group to galaxy $\mathcal{M}/M \sim 10^1$, estimates borrowed from collapse theories yield dispersion ratios $V/v \sim (\mathcal{M}/M)^{1/12} \sim 1$ in systems formed from initial fluctuations with a white-noise power spectrum, as often used in simulations. Other power spectra $\langle |\delta_k|^2 \rangle \propto k^n$ still give $V/v \sim (\mathcal{M}/M)^{(1-n)/12} \sim$ a few for $n \gtrsim -2$. In general, the weak dependence $V/v \sim (\mathcal{M}/M)^{1/3}(\rho_a/\rho)^{1/6}$ holds. The actual value of V/v easily falls under the previous estimates, as the galaxies specifically gained contrast from dissipative formation; in addition, the internal degrees of freedom are heated up by the interactions themselves.

Our main case FI may be compared with other kinds of interactions having $\lambda < 1$, e.g., the geometrical limit (GI) having cross section $\Sigma \approx \eta\pi(r + r')^2 \propto (m^{1/3} + m'^{1/3})$, with $\lambda = \frac{2}{3}$ and efficiency $\eta \lesssim v^2/V^2 \ll 1$. Here the merging rate reads $\tau^{-1} \propto \eta\rho_a V/\rho^{2/3} M_*^{1/3}$, so that not only the initial growth is slower, but also any attempt to accelerate the rate $\tau^{-1} \propto M_*^{-1/3}$ by the t-dependent coefficient is counteracted by the very growth of $M_*(t)$.

4. ANALYSIS OF THE RUNAWAY: THE MASS DISTRIBUTION

Beyond the heuristics, the structure of equation (2.1) contains a novel phenomenon. We derive analytic solutions of equation (2.1) that scale—after an initial transient—as $N(M, t) \to M_*^{-x}(t)\phi(m)$, equivalent in normalized form to $(\mathcal{M}/M_*)^{x-2}\phi(m)/M_*^2$. When $x = 2$ holds and ϕ has an upper cutoff, this form plainly conserves the system mass $\mathcal{M}$ at all times.

In general, each side of equation (2.1) separates into a t-dependent and an m-dependent factor. The time-dependent factors must balance leading to the equation

$$\frac{dM_*}{dt} M_*^{-\lambda}\left(\frac{M_*}{\mathcal{M}}\right)^{x-2} = \mathcal{F}(t) , \tag{4.1}$$

which constitutes the exact counterpart of the estimate in equation (3.1). In turn, the factors depending on m must satisfy the other equation:

$$m\,\frac{d\phi(m)}{dm} + x\phi(m) = \phi(m) \int_0^{m_{\max}} dm'\psi(m, m')\phi(m') - \frac{1}{2}\int_0^m dm'\psi(m', m - m')\phi(m')\phi(m - m') , \tag{4.2}$$

that governs the shape $\phi(m)$ of the mass distribution under mergings.

For $m \gg 1$ a detailed analysis (see Ernst 1986) of the right-hand side of equation (4.2) yields

$$\phi(m) \to m^{-\lambda}e^{-m} \tag{4.3}$$

for a generic homogeneity degree λ. For $m \ll 1$, it is seen that when $\lambda > 1$ holds, and specifically for FI, the solution is finite and of the form

$$\phi(m) \to m^{-x} , \tag{4.4}$$

with x related to λ as we show next. The asymptotic shapes of the mass distribution play an important role in the evolution of the system, which is best understood considering the mass flux (see van Dongen & Ernst 1985) across $m_{\max}$

$$\dot{\mathcal{M}} = -\left(\frac{\mathcal{M}}{M_*}\right)^{2x-4} M_*^{\lambda-1}\mathcal{F}(t) \int_0^{m_{\max}} dm' \times \int_{m_{\max}-m'}^{m_{\max}} dm''m'\psi(m', m'')\phi(m', t)\phi(m'', t) . \tag{4.5}$$

When $M_{\max}$ falls in the range of m where already the form (4.3) applies, the exponential cutoff plainly yields $\dot{\mathcal{M}} = 0$, implying conservation of total mass. But an interesting situation arises when $M_{\max}$ falls in the range where still $m \equiv M/M_* \ll 1$ holds, and hence formula (4.4) instead applies. This requires the characteristic mass to run away, i.e., technically $M_* \to \infty$ at a *finite* time t_∞, as proved in § 5 for $\lambda > 1$. Given this, the result is $\dot{\mathcal{M}} \propto -m^{3+\lambda-2x}M_*^{3+\lambda-2x}$. The condition for $\dot{\mathcal{M}}$ to be $\neq 0$ and *finite* is

$$x = (\lambda + 3)/2 > 2 , \tag{4.6}$$

namely, $x = 13/6$ for our case FI with $\lambda = 4/3$. We require a finite $\dot{\mathcal{M}}$ for consistency with a time-resolved description, stable relative to different values of the system mass.

The meaning of $\dot{\mathcal{M}} < 0$ is as follows. On approaching the divergence at t_∞, the mass distribution is rapidly stretched out into a power-law tail by the increasing $M_*(t)$. This triggers a finite mass flux $\dot{\mathcal{M}} < 0$ from the set of normal galaxies described by equation (2.1). By global mass conservation, the mass leaving this set must flow into a second phase. Correspondingly, the system mass breaks up into a bimodal distribution with an increasing separation; see Figure 1. The first phase is constituted by the normal galaxies with a steep distribution $\phi \propto m^{-(\lambda+3)/2}$. The second phase is constituted by the forming merger, with zero number density but a finite mass, actually growing at the rate $|\dot{\mathcal{M}}|$.

5. THE CHARACTERISTIC MASS

We now examine equation (4.1) with the value of x given by equation (4.6), to find the conditions for an actual divergence of $M_*(t)$.

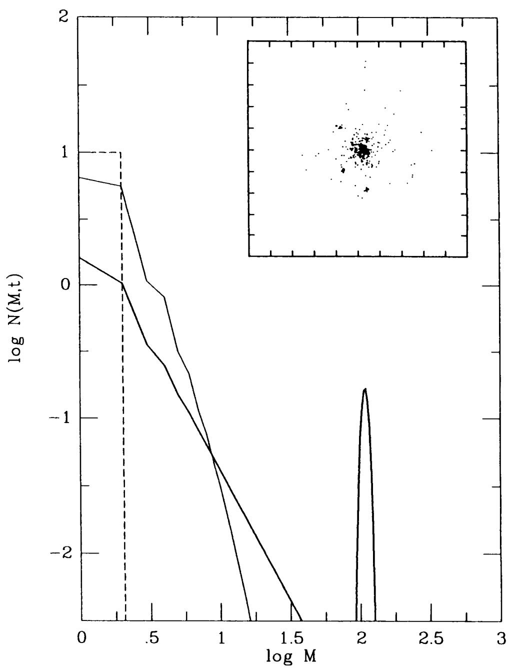

FIG. 1.—The shapes of the mass distribution of normal galaxies for FI ($\lambda = 4/3$) in the premerger ($t = 1.5t_0$, *thin curve*) and in the postmerger ($t = 6t_0$, *thick curve*) stage, as resulting from numerical *integrations* of eq. (2.1) from the initial condition shown at extreme left. The slope of the postmerger distribution is $x \approx 2.15$, to be compared with the value 13/6 evaluated in § 4. It is also represented by the merger with mass $\mathcal{M}(0) - \mathcal{M}(t)$. The inset reproduces a typical postmerger configuration from the N-body *simulations* of Carnevali et al. (1981). Units: $10^{11}\ M_\odot$ and 1 Mpc.

The function $\mathscr{F}(t)$ on the right-hand side is determined up to a separation constant set with the use of the zeroth and second moments of $N(M)$ (Ernst 1986) and contains all the time dependences in the kernel. We parameterize its t-dependence as $\mathscr{F}(t) = F_0(t/t_0)^f$, with the following breakup: the ambient density increases sharply at recollapse, and during virialization and thereafter may be described by $\rho_a(t) \propto t^d$ with $d \sim \frac{2}{3}$. The internal density is poorly known, and we let $s \sim 0$–d in the parameterization $\rho \propto t^s$. Finally, $V(t)$ increases weakly, as said, and we set $V(t) \propto t^u$ with $u \lesssim \frac{1}{3}$. In our case FI, $f = d - s/3 - u \sim \frac{1}{3}$–0 holds.

Then equation (4.1) is solved by (see Fig. 2)

$$M_*(t) = M_*(t_0)\left\{1 - k\left[\left(\frac{t}{t_0}\right)^{f+1} - 1\right]\right\}^{2/(1-\lambda)} \propto \left[\left(\frac{1+k}{k}\right)t_0^{f+1} - t^{f+1}\right]^{2/(1-\lambda)}, \quad (5.2)$$

where $k \sim n_0 \Sigma R(\mathscr{M}/M_{*0})^{(1+\lambda)/2}(\lambda - 1)/2(f + 1)$ includes the separation constant.

It is seen that when $\lambda > 1$ (as applies for FI) and for $f > -1$ (requiring no or slow expansion), indeed $M_*(t)$ formally diverges at a *finite* time, which proves the *runaway*. In fact, for FI this takes only some dynamical times because the outer exponent in equation (5.1) takes on the value $2/(1 - \lambda) = -6$, and $t_\infty = t_0[(1 + k)/k]^{1/(f+1)} \lesssim 5t_0$ holds; the initial time t_0 stands for the epoch $t \sim t_d$ when a typical density perturbation turns around to collapse and form the group.

So the argument closes up for $\lambda > 1$, and specifically for $\lambda = 4/3$. By way of contrast, for $\lambda < 1$ (as is the case for GI) the solution $M_*(t)$ increases monotonically yet remains finite at any t (see Fig. 2), because the outer exponent becomes $1/(1 - \lambda) > 0$. Then we cannot expect $\dot{\mathscr{M}} \neq 0$; instead, the mass distribution evolves self-similarly and slowly, direct mergings

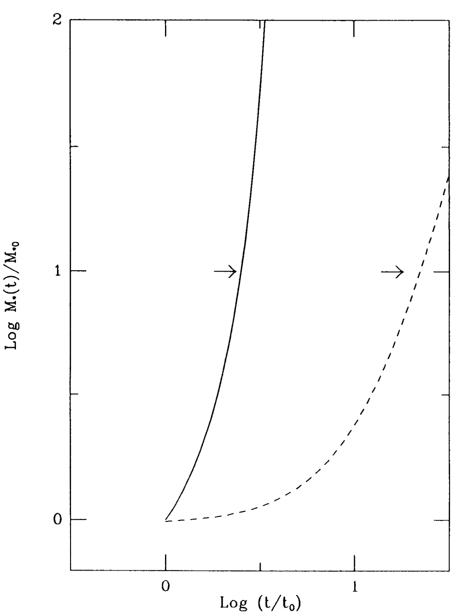

FIG. 2.—The characteristic mass $M_*(t)$ is compared for interactions of the FI kind ($\lambda = 4/3$) and of the GI kind ($\lambda = \frac{2}{3}$). We represent by the continuous curve the FI. The dashed curve refers to the maximal GI, computed with $\eta = 1$, and with $\rho_a \propto t^{2/3}$ which actually describes the overall contraction associated with a true runaway regime. Even so, an increase of M_* by a factor of 10 takes times longer by a factor ~ 10.

being ineffective to produce a large merger over dynamical time scales.

Figure 1 also presents preliminary results from numerical solutions of equation (2.1) for $\lambda = 4/3$ in the premerger and postmerger stage, that bear out our analysis remarkably.

6. CONCLUSIONS AND DISCUSSION

In self-gravitating galaxy groups the shape and the evolution of the galaxy distribution $N(M, t)$ are driven by interactions to become *independent* of initial conditions on a scale provided by a multiple of the dynamical time set at system collapse.

Focused resonant interactions drive the *runaway* formation of a large merger that eventually gobbles most mass in the system. The critical phenomenon is the onset of mass loss from the set of normal galaxies described by equation (2.1). The system breaks up into two phases: (1) the set of normal galaxies with a steep mass distribution dominated in number and mass by small, least interacting objects; and (2) a merger at the upper mass end, gaining mass from the former. This merging process is a form of gravitational runaway, with time scale given by equation (3.1), equivalent to $\tau \sim V/rG\rho_a \sim (\rho/\rho_a)^{1/2}(G\rho_a)^{-1/2}$.

With $\lambda > 1$, runaway pace given by $\dot{M}_* M_*^{x-\lambda-2} \propto t^f$, and slope of the residual distribution $\phi \sim m^{-x}$ are *related* by $x = (3 + \lambda)/2 > 2$ in terms of the cross section scaling $\propto M^\lambda$, by the key requirement of a transition time-resolved and independent of the finite system mass. The interesting range is $4/3 \leq \lambda \leq 3/2$. We have discussed the case $\lambda = 4/3$. The other end is $\lambda = 3/2$, resulting from the scaling $r \propto M^{1/2}$ as given by the observational Faber-Jackson relation $L \propto v^4$ (Faber 1982) at $M/L \sim$ const. Because this also satisfies the key condition $\lambda > 1$ it yields very similar outcomes, actually with a shorter t_∞.

Our results for FI in a *finite* system are formally analogous, yet not identical, to the phase transition sol to gel ("gelation") occurring in infinite suspensions of aggregating particles (see Ernst 1986). The fully developed *merging runaway* constitutes the first neat instance of a *phase transition* of gravitational nature in the relatively nearby universe. The appropriate order parameter is constituted by the normalized merger mass $1 - \mathscr{M}(t)/\mathscr{M}(0)$.

The runaway is eventually *stabilized* as resonant interactions are quenched by the decrease in number and size of the surviving galaxies. These are small also because the grazing interactions peel off the least bound external regions. The residual interactions are best described as dynamical friction, with asymmetrical cross sections (see Alladin, Narasimham, & Ballabh 1988), of satellites with large angular momenta.

Two extreme regimes may be envisaged: for $V/v \sim 1$ resonant interactions dominate and the runaway proceeds; when $V^2/v^2 \gg 1$ initially, the evolution is self-similar and slow, liable to an early termination by inclusion of the system in a still larger cluster.

Initial presence of much diffuse dark matter (as opposed to individual galaxian halos) will stretch out the merging times, as then V^2 is enhanced at given luminous mass. Large membership also concurs, by interference of interlopers with the two-body collisions (Mamon 1990). In rich clusters the velocity dispersion is large enough to suppress the direct mergings, making it difficult to build up a cD body inside rich clusters (Carnevali et al. 1981; Merrit 1983; Richstone & Malumuth 1983; Bothun & Schombert 1988). In such environments, a slower form of merging may prevail (see Richstone 1990): first, dynamical friction segregates the galaxies to the center, then merging or cannibalism take place.

Our analytic findings closely *match* the average results from N-body experiments recalled in § 1 as for quantitative time scales and for morphologies, as visualized by Figure 2. The experiments can start from such special phase-space structures as large angular momenta which slow down mergings (Governato, Bhatia, & Chincarini 1991); in our analysis, these will be embodied in a reduced form, through an efficiency $\eta < 1$.

The *reality* of runaways gone to near completion is supported by such groups dominated by a cD-like galaxy (e.g., MKW 11, AWM 4, AWM 7) as cataloged by Morgan, Kayser, & White 1975; Albert et al. 1977). Indirect evidence is provided by X-ray emission from groups (cf. Schwartz, Schwartz, & Tucker 1980; Biermann et al. 1982), since extensive mergings induce shrinking of the overall configuration so increasing the density of the intergalactic gas, which boosts and sustains the bremsstrahlung emissivity $\propto n^2$ over several dynamical times.

Our solutions of equation (2.1) *differ* sharply from the fully self-similar solutions (see Silk & White 1978) forced to evolve on a cosmic time scale by imposing the condition $\tau \propto t$. In a critical universe, the latter kind would be induced in the "field" by the fast expansion $\rho_a \propto t^{-2}$, such that the scale $\tau \sim 1/(G\rho_a)^{1/2} \rightarrow t$. In an open universe the even faster expansion eventually freezes the evolution since $\tau/t \rightarrow \infty$. We will discuss elsewhere in full the relationships between these branches of the hierarchical clustering.

Here we only note that *variants* of the merging runaway are likely to operate also in environments other than spherical, long-lived groups. First, redshift surveys (e.g., Ramella, Geller, & Huchra 1989; Sutherland 1988), large-scale simulations (Efstathiou et al. 1988; Villumsen 1989), and quasi-linear analyses (see Shandarin & Zel'dovich 1989), all stress structures with dimensionality $D < 3$ and correspondingly reduced expansion rates that allow some merging activity. Second, in many groups with short survival time and/or large V^2/v^2, the runaway will begin only to be prematurely terminated at the stage of building large ellipticals possibly with stimulated starbursts.

Such milder and more common variants of the merging action will be relevant to the deep galaxy counts (Tyson 1988; Cowie et al. 1990; Koo 1990; Guiderdoni & Rocca-Volmerange 1990) for assessing the role of number evolution.

We are indebted for helpful discussions to L. Biferale, F. Lucchin, S. Matarrese, N. Vittorio, and especially to M. Vietri, and thank the referee for constructive comments.

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