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Letter to the Editor

Distortions of the CMB spectrum by distant clusters of galaxies

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Abstract The distribution in cosmic depth of clusters and groups of galaxies can be probed by means of the Sunyaev-Zel'dovich effect from the associated intra-cluster plasma. The average spectral distortions predicted, from observations of individual clusters, by representative cosmogonies are computed and discussed in the perspective provided by the current generation of space instrumentation. Significant observational bounds to the cluster age may be within the reach of COBE.

Key words: cosmology – background radiations – clusters: of galaxies

1. Introduction

Clusters and groups of galaxies provide a clear testing ground for cosmogony. These are the largest structures observed to be in gravitational equilibrium (contrasts $\gtrsim 10^2$ relative to the background ρ_u), and were conceivably formed with little or no dissipation (Rees & Ostriker 1977). Back to what redshift shall we find them settled?

The present informations are apparently conflicting. On the one hand, Gunn and collaborators (see Gunn 1990) observe some clusters with high velocity dispersions at redshifts $z \sim 0.7$ and beyond. On the other hand, Frenk et al. 1990 and Olivier at al. 1990 argue that apparent richness and dispersion of distant clusters are enhanced by projection effects.

X-ray surveys can, in principle, single out virialized potential wells of sizes $R_e \sim$Mpc and masses $M \sim 10^{13} \div 10^{15}\ M_\odot$ out to $z \sim 0.5 \div 1$. They take advantage of the powerful, extended X-ray emission (Cavaliere et al. 1971, Gursky et al. 1972) from the hot intracluster plasma (ICP) with central densities $n_c \sim 10^{-3}\mathrm{cm}^{-3}$ and large masses $M_{ICP} \sim 10^{-1}M$, heated to virial temperatures $kT_P \sim GMm_H/10R_e \sim$ keV. Moreover, X-ray luminosities are less liable to contaminations from projection effects, being highly non-linear with M. However, the outcome of the first such survey (Gioia et al. 1988) has raised controversy. While Cavaliere & Colafrancesco 1988 interpreted the resulting, flat number counts as evidence for a dearth of luminous distant sources, others (e.g., Pesce et al. 1990) argue for flux miscalibrations and cooling flows making them consistent with a distribution uniform in depth.

So alternative means to probe the space distribution of these structures are important. One such probe is again provided by the physics of the ICP. This not only implies copious bremsstrahlung emission in X-rays, but also distortions in the spectrum of cosmic microwave background (CMB), as the cold photons on crossing the clusters are weakly Comptonized by the hot electrons (Sunyaev & Zel'dovich 1972). Thus line of sight superpositions can be put to constructive use.

The Sunyaev-Zel'dovich effect (SZE) has been reliably measured at high radio frequencies for 3 individual clusters, to find apparent diminutions of the CMB temperature $\Delta T \sim -1/2$ mK resolved over several arcminutes. These clusters may constitute the high-richness tip of a distribution, but ought not to exceed by more than a factor ~ 5 the value expected from all rich clusters (Birkinshaw 1990). Theoretically (Rephaeli 1981, Cavaliere et al. 1986), we expect from individual rich clusters a SZE of order $|\Delta T/T| \sim 10^{-4\pm1/2}$.

The SZE due to the summed contribution of distant clusters has two facets: spectral distortion and angular fluctuations, to some extent complementary. The former requires absolute measurements, with a beam wide compared with the cluster angular diameters. It is sensitive to the integrated distribution of remote and reasonably sized structures, but it may be confused by the presence of diffuse intergalactic plasma.

Recently the FIRAS instrument on board the COBE – with only 9 minutes worth of data over its 7^o aperture – has set a bound $y \lesssim 10^{-3}$ to the Comptonization parameter from hot plasma of any origin (Mather et al. 1990). The bound is low enough to seriously challenge the combination of diffuse plasma density and temperature $\Omega_B\ T \sim 2.5\ 10^8$ K required to make, at relatively high z, a dominant contribution to the hard X-ray background (XRB). Accumulated data may lower the bound on y by an order of magnitude, *provided* the systematic uncertainties (mainly related to calibration uniformity) turn out to be lower yet.

With the XRB from diffuse hot plasma in serious doubt, it is hard to envisage other evidence, or astrophysical processes related to hierarchical cosmogonies, implying $y > 10^{-4}$ (see also Cen et al. 1990). In fact, dynamical temperatures ought to be bounded by $T < 10^8$K, the virial value in the cluster wells. Deeper wells might be provided only by presumably young and local giant superclusters (Scaramella et al. 1989). The baryon density is bounded, after reappraisals of cosmic nucleosynthesis, by $\Omega_B \lesssim 0.1$ (Steigman 1990). The impact on the intergalactic baryons of AGN emissions explaining themselves the XRB produces $y \ll 10^{-4}$, see Collin-Souffrin 1990.

It is then interesting to investigate the integrated effect of the hot plasma known to reside in high-contrast clusters, and expected to compound into a curtain thickening along our line of sight. So in the perspective provided by the data gathered by COBE, we proceed from single cluster observations to evaluate the predictions by cosmogonic scenarios spanning the interesting range.

2. Framework

An individual cluster produces in the Rayleigh-Jeans region an apparent diminution of the background radiation temperature T given, along a single line of sight, by

$$\frac{\Delta T}{T} = -2\,y = -2\,\frac{k\sigma_T}{m_e c^2}\int d\ell\, n(r)\, T_P \qquad (2.1)$$

in terms of the electron density n and temperature T_P. The spectral distortion $\langle \Delta T/T\rangle$ averaged over a wide beam builds up along the line of sight in terms of the single cluster's effect multiplied by the geometrical optical depth $d\tau \sim dl\; dM\; N(M)\;\pi R^2$, where R is the cluster radius and $N(M)$ the number density per unit mass range.

Since $\Delta T/T \propto n\,R\,T_P$ for a single object and $T_P \propto M/R$, the scaling $\Delta T/T \propto n\;M$ follows, while $R \propto (M/\rho)^{1/3}$ also holds in terms of the internal mass density ρ. Thus the rate of increase of the average SZE along the line of sight reads

$$d\,\langle \Delta T\rangle/dl = A(z)\int dM\,N(M)\,M^{5/3}\;. \qquad (2.2)$$

The factor $A(z)$ scales as $n(z)\;\rho_u(z)\;[M_c(z)/\rho(z)]^{2/3}$, considering a generic self-similar and mass-conserving distribution $N(M,z) \propto \rho_u/M_c^2$ that cuts off for $M > M_c$.

The scaling of M_c with z depends on cosmogony. For the uniform distribution (UD), M_c is constant by definition.

Canonical Hierarchical Cosmogonies (HCs), instead, hold the virialized objects to form by direct collapse of sequentially larger masses (on average) from density perturbations that have a Fourier power spectrum $\langle|\delta_k|^2\rangle \propto k^\nu$; the index ν may be either fixed at the phenomenological value $\nu = -1.2$, (Peebles 1980) or varying with M, e.g., after the CDM model (Peebles 1982). In these cosmogonies, the internal density at virialization $\rho(z)$ scales as $\rho_u(z)$, with n/ρ often taken constant, while the typical mass virializing at z to form structures with density contrast $\rho/\rho_u \sim 180$ follows in a FRW critical universe $M_c \propto (1+z)^{-1/a}$ with $a \equiv (\nu+3)/6$.

Thus at higher z the HCs envisage more and denser objects, but cooler and smaller (the typical condensations were then poor clusters or even groups) such as to give decreasing individual contributions to the SZE. These features drive opposite trends, whose balance depends upon the specific perturbation spectrum being considered.

For canonical HCs the simple expression $\langle \Delta T/T\rangle \propto \int dl\,n\,T_P$ applies, involving average plasma densities at mean virial temperatures. In a critical universe, where $dl \propto dz/(1+z)^{5/2}$, the scaling reads $(1+z)^{\frac{(7+5\nu)}{2(\nu+3)}}$. When $\nu > -1.4$ this increases, if slowly, for $z > 1$. For $\nu < -1.4$ (as holds for small masses in the CDM model) it starts from similar local values to converge soon after $z \gtrsim 1$. The UD, instead, yields simply $(1+z)^{3/2}$, or $(1+z)^2$ in an empty universe.

We normalize the numerical coefficients in eq. (2.2) to the information coming from observations of individual clusters. As to the ICP state, we focus on the approximate isothermal consistent with the X-ray data (see Hughes 1989) and indicated by various N-body simulations (Perrenod 1978, Evrard 1990): given enough time, the electron temperature T_P approaches the virial value behind a shock expanding out to radii $\gg r_c$, the "core"radius or rather the inner scale of the potential.

We use the isothermal decrease of $n(r)$ from the cluster center given by Cavaliere & Fusco-Femiano 1976, to find at an offset x from the central line of sight

$$\frac{\Delta T}{T}(x) = \left(\frac{\Delta T}{T}\right)_c \frac{2}{\pi[1+(x/r_c)^2]^{3\beta/2-1/2}} \times$$

$$tan^{-1}\left(\frac{R^2-x^2}{r_c^2+x^2}\right)^{1/2}. \qquad (2.3)$$

The observations are conveniently reduced (cf. Birkinshaw 1990) to the the central value $\left(\Delta T/T\right)_c \propto T_c\; n_c\; r_c$ which we scale down to $0.7\;10^{-4}$ (implying $n_c \sim$ a few 10^{-3}) for the Abell (1958) richness class 1, more typical of the local clusters. To deal *conservatively* with the broadness of the SZE, we consider values $2/3 \le \beta < 1$, and represent the shock with a sharp cutoff in the run of T (and/or n) at R close to the virial radius.

Integrating eq. (2.3) out to an effective radius R_e, we obtain for the average $(\Delta T/T)_e$ the value $(\Delta T/T)_e = (r_c/R_e)^{3(1-\beta)}\; 2/3(1-\beta)(\Delta T/T)_c$ for $R_e < R$. For $R_e = R$ and $\beta = 2/3$ we find $(\Delta T/T)_c = (4r_c/\pi R)[1 - (r_c/R)\;tan^{-1}(R/r_c)]\;(\Delta T/T)_c$.

To cover the ICP in groups (for X-ray evidence see Bahcall et al. 1984), we adopt as representative the mass distribution $N(M,z)$ of Press & Schechter (1974) that is known to fit both the (scanty) data and the results of N-body simulations (Frenk et al. 1990), with a bias parameter $1 \le b \lesssim 1.5$ plugged in (see Peebles, Daly and Juszkiewicz 1989 for a discussion). To gauge different cosmogonies, this will be written in terms of $m \equiv M/M_c$ as

$$N(m,z) \propto \rho_u(z) M_c^{-2}(z)\, m^{a-2}\, e^{-b^2 m^{2a}/2}\;. \qquad (2.4)$$

This form exhibits the self-similar, strong evolution for $z > 0$ of the HCs, with $M_c(z)$ decreasing rapidly. The normalization is computed as to match the observed local value $N_{obs} \simeq 1.3\,10^{-6}\,h^3\,Mpc^{-3}$ for the density of richness $r \ge 1$ clusters (Ramella et al. 1990, Scaramella et al. 1991; $h \equiv H_o/50$ km s^{-1} Mpc^{-1}).

Now we integrate eq. (2.2) over masses and z to find

$$\langle\frac{\Delta T}{T}\rangle = B\; N_{obs}\;\pi\, R_e^2\;(\frac{\Delta T}{T})_e\; c\; F(z)/H_o \qquad \text{with} \qquad (2.5)$$

$$B \equiv \frac{\Gamma(\frac{1}{2}+\frac{1}{3a})(\frac{2}{b^2})^{5/6a}}{\Gamma\left(\frac{1}{2}-\frac{1}{2a},\,\frac{b^2}{2}\right)},$$

$$F(z) \equiv \int_0^z dz\;\frac{(1+z)}{(1+\Omega_o z)^{1/2}}\;[\frac{M_c(z)}{M_c(0)}]^{2/3}\;[\frac{\rho(z)}{\rho(0)}]^{1/3}.$$

The coefficient B takes on values 8.6, 6.1 for $b = 1,\; 1.5$, respectively, with $a = 0.3$. Our normalization to the observed $\Delta T/T$ leaves no explicit dependence on h, but cosmology enters through the kernel and $M_c(z|\Omega_0)$. The latter depends also on cosmogony, which enters explicitly through a, b.

3. Results

For $\Omega_0 = 1$, we normalize eq. (2.5) to $r_c = 0.35\ h^{-1}$ Mpc, $R = R_e = 2.4\,h^{-1}$ Mpc, $\beta = 2/3$, and $b = 1.5$ to obtain for the isothermal ICP the results below. These scale as $(\Delta T/T)_c\ r_c\ R_e$, and for $b = 1$ increase by 1.4.

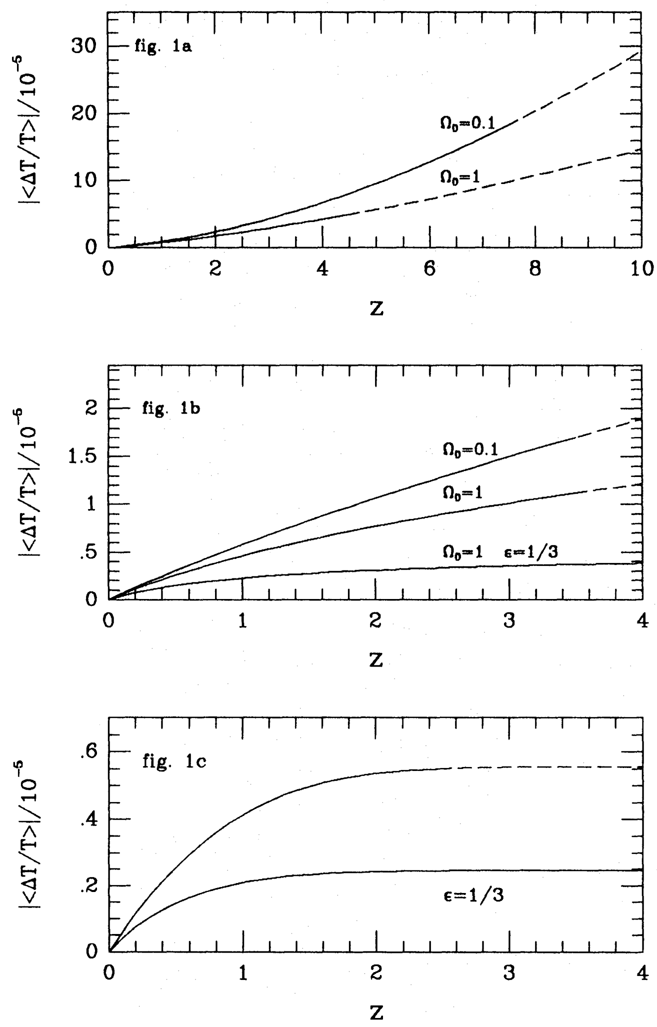

Fig. 1a. The apparent diminution of the CMB temperature in the R.J. region caused by clusters uniformly distributed out to a redshift z, after eq. (3.1) with parameters given at the beginning of Sect. 3. For $b = 1$ the values of $|\langle\Delta T/T\rangle|$ increase by 1.4. Continuous lines end at the limits set by Compton cooling in the absence of continuous energy input ($\Omega_0 = 0.1$), or by formation times ~ 1 Gyr ($\Omega_0 = 1$).
Fig. 1b. The apparent diminution after hierarchical clustering with $\nu = -1.2$ and $\Omega_0 = 1$, see eq. (3.2). For $\Omega_0 = 0.1$ the appropriate $M_c(z|\Omega_0)$ is referred to in Sect. 3. Limits are set by formation times for groups. The lower curve refers to the case of an ICP relative content proportional to M^ϵ with $\epsilon = 1/3$.
Fig. 1c. As above, for a CDM initial spectrum with $\Omega_0 = 1$.

• *Uniform distribution*: Here M_c, ρ, n =const, so that

$$\langle\frac{\Delta T}{T}\rangle = -0.4\,10^{-5}\ [(1+z)^{3/2} - 1]\ , \qquad (3.1)$$

see fig. 1a. Note $\langle\Delta T/T\rangle \sim \left(\Delta T/T\right)_e$ for a geometric optical depth $\langle\tau\rangle \sim 1$, as it should for marginal overlap.

• *Hierarchical Cosmogonies with ν = constant*: Here $M_c \propto (1+z)^{-1/a}$. For $\nu = -1.2$ we find

$$\langle\frac{\Delta T}{T}\rangle = -2.3\ 10^{-5}\ [(1+z)^{0.27} - 1]\ , \qquad (3.2)$$

see fig. 1b. Instead, for a white noise spectrum with $\nu = 0$ we find

$$\langle\frac{\Delta T}{T}\rangle = -0.6\ 10^{-5}\ [(1+z)^{7/6} - 1]\ . \qquad (3.3)$$

All these runs branch off from the same small-z behaviour.

• *Cold Dark Matter, $\nu = \nu(M)$*: Here ν, and hence a, decrease with M (depending on h^2) and $M_c(z)$ scales down fast with z. The result of a numerical integration is plotted in fig. 1c, which shows convergence for $z \gtrsim 2$ to the asymptotic value $\langle\Delta T/T\rangle = -0.6\ 10^{-5}$.

For $\Omega_0 < 1$ the changes concern the kernel of $F(z)$ in eq. (2.5), and $M_c(z|\Omega_0)$ now taking a slower run, cf. White & Rees 1978. The resulting behaviours of $\langle\Delta T/T\rangle$ are shown in figs. 1a, 1b for the UD the HCs, respectively.

Smaller values for Ω_0 imply stronger distortions of the CMB, as a given angular area corresponds to larger effective volumes, with $N(M,\ z)$ closer to a UD. The UD itself in low density universes yields *upper* bounds to the CMB distortions, constraining the evolutionary time scales of either the universe or the structures.

At the opposite extreme, we consider time change of the ICP content. X-ray data (David et al. 1990) indicate the ratio of ICP to stellar barions to increase from values ~ 1 in groups to ~ 5 in rich clusters, which may be interpreted as an effect of the average formation epoch (Cavaliere & Colafrancesco 1990). This may be parameterized as $n/\rho \propto m^{1/3}\ M_c^{1/3}(z)$, which *includes* the galaxy contributions (see Trester & Canizares 1989). The integration over z will now give even smaller contributions, as shown in figs. 1b and 1c by the lowest curves. Very low limits set to spectral distortions will imply such ICP anti-evolution.

4. Discussion

The release of gravitational energy heats up progressively the diffuse barions in larger cosmic structures, and preserves a high level of thermal energy density that can be probed by means of $\langle\Delta T/T\rangle$ over a long line of sight. In the limit of long survival time, with the local structure extending uniformly into the past, *upper* bounds $|\langle\Delta T/T\rangle| \sim$ a few 10^{-4} arise, corresponding to $\langle\tau\rangle$ ~few. Such values could be probed by COBE with improvement by an order of magnitude over the initial systematic uncertainties (Mather et al. 1990).

By providing bounds $y < 10^{-4}$, instruments of the FIRAS generation would set correspondingly strong bounds to the simple UD. This, in the consistent framework of a low-density universe with $b \sim 1$, shall not be continued to its natural limits set rather fuzzily by physical overlap, or by formation time $t_{dyn} \lesssim t$, or most stringently by Compton cooling of the ICP (see fig. 1a), rather it will have to be terminated ad hoc at $z < 6$. For $\Omega_0 = 1$ the limit is close to that set by dynamical times. Instead, stronger limits obtain for Lemaitre universes with $\Lambda \sim \Lambda_E$ where formation times are less constraining. The bounds are *conservative*: high temperatures beyond the virial radius might yet enhance the distortion. In sum, the SZE will provide a direct *observational* probe of the *age* of the local rich clusters; in a longer perspective, $y < 10^{-5}$ will imply independently of Ω_0 formation later than $z \sim 2$.

All HCs predict smaller and cooler structures into the past, and yield smaller distortions. The strong evolution expected for $N(M, z)$ in the canonical version of such cosmogonies (direct collapses driven by the density perturbation field), gives $\langle \Delta T/T \rangle \sim -1\,10^{-5}$ with linear spectral index $\nu \simeq -1.2$ and ICP content constant up to $z \sim 3.5$. A still lower result, $\langle \Delta T/T \rangle \sim 0.6\ 10^{-5}$, obtains with a CDM spectrum because $M_c(z)$ decreases so fast (here CDM has been forced to the local structures, wich may overestimate its effectiveness). These values, even more so with an ICP content decreasing into the past, lead to *lower* bounds hardly probed with the present generation of space instrumentation. We shall discuss elsewhere the contributions from lower-constrast structures on their way to collapse.

In general, the observed results will depend on total evolution: the *slower* is this in a given *cosmological* and *cosmogonic* scenario, the *stronger* is the distortionn of the CMB spectrum.

Included in cosmogony is the history of diffuse barions, as summarized in the z-dependence of n/ρ. But the SZ measurements offer several possibilities for *intercalibrations*. First, the spectral distortion $\Delta T/T$ measured in the R-J region provides the value expected in the Wien region: e.g., at $\lambda \simeq 600\mu m$ we have a *positive* $\langle \Delta I/I \rangle = -2.8\ \langle \Delta T/T \rangle_{RJ}$, a sign change which constitutes a characteristic signature of the SZE (Sunyaev & Zel'dovich, 1972) at wavelegths clear from dust contamination.

In addition, spectral measurements can calibrate n/ρ or M_{ICP}/M so providing information relevant to the other facet of the SZE, the angular fluctuations: these are sensitive to irregularities in the patchy cluster curtain between us and the last scattering surface, but independent from any uniform veil of plasma. Cole & Kaiser (1988) have evaluated the angular fluctuations in the canonical CDM scenario with $\Omega_0 = 1$, to find $\langle (\Delta T/T)^2 \rangle^{1/2} \simeq 5 \times 10^{-6}$ on angular scales of 7' with normalizations close to ours. For this specific model, which favors rich clusters at fairly recent epochs, we find a spectral distortion $\sim -1\ 10^{-5}$, while in the HC scenario with $\nu = -1.2$ we find distortions enhanced by ~ 2 and fluctuations by somewhat less: fluctuation measurements with the accuracy of Readhead et al. (1989) are really close to be telling.

Finally, the SZ measurements intercalibrate with the X-ray fluxes which depend on $(M_{ICP}/M)^2$, as discussed by Cavaliere, Danese and De Zotti 1979. In fact, actual detections will rise the exciting possibility to bypass the ambiguities affecting any given spectral band, and even to calibrate out the ICP content, leaving cosmogony $\times$ cosmology fully exposed. Once the cosmogony is known, the ICP history can be recovered from observations in a single band.

We expect a combination of such measurements to elicit any early dynamical structures containing hot plasma. If, on the other hand, the bounds on SZ fluctuations will turn out to be low, in return the minute wrinkles of the last scattering surface will stand out unveiled.

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