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VELOCITY BIAS FROM MERGING IN CLUSTERS OF GALAXIES: THE $\beta < 1$ PROBLEM

R. FUSCO-FEMIANO
Istituto di Astrofisica Spaziale, CNR, C.P. 67, I-00044 Frascati, Italy
AND
N. MENCI
Osservatorio Astronomico di Roma, via dell'Osservatorio, I-00040 Monteporzio, Italy
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ABSTRACT

We study the evolution of the galaxy velocity distribution in galaxy clusters under binary aggregations. Starting with an initial Maxwell distribution, we solve the complete Boltzmann-Liouville equation including collisions. We find an asymptotic distribution characterized by a galaxy velocity dispersion smaller than that of the dark matter. This is due to the transfer from orbital to internal energy occurring in galaxy merging, which is not completely balanced by the galaxy response to the cluster gravitational field. As a consequence, the value of the parameter β that enters in the standard hydrostatic isothermal β-model is less than 1, as determined from the fits to the X-ray surface brightness data. The result is robust with respect to different shapes of the cluster mass distribution. The dependence of β on the cluster velocity dispersion and size is computed and discussed.

Subject headings: galaxies: clusters: general — galaxies: distances and redshifts — X-ray: galaxies

1. INTRODUCTION

Much observational evidence (Kent & Gunn 1982; Kent & Sargent 1983) indicates that the galaxy velocity distributions in regular clusters of galaxies are in a relaxed quasi-stationary state, believed to be reached by "violent relaxation" during cluster formation. In this case, the equilibrium state can be derived as the most probable phase-state configuration that conserves energy, momentum, and particle number. In the absence of galaxy collisions, this state is characterized by a Maxwell-Boltzmann distribution (Lynden-Bell 1967) where the velocity dispersions of the galaxies, $\langle v^2 \rangle^{1/2}$, and of the dark matter particles, σ, are the same and are given by the virial theorem.

The most stringent test of this picture comes from cluster X-ray observations. In fact, the X-ray emission is due to a diffuse intracluster gas which is in equilibrium with the cluster potential wells and emits by thermal bremsstrahlung (Cavaliere, Gursky, & Tucker 1971; Solinger & Tucker 1972). Thus, the gas temperature T is a direct probe of the cluster potential and can be directly compared with the galaxy velocity dispersion.

This comparison is performed in terms of the parameter $\beta = \mu m_p \langle v^2 \rangle / 3kT$ (μ is the mean molecular weight, m_p is the proton mass, k is the Boltzmann constant, and $\langle v^2 \rangle/3$ is the line-of-sight velocity dispersion), which enters in the hydrostatic isothermal β-model (Cavaliere & Fusco-Femiano 1976) used successfully to fit the X-ray surface brightness emission from numerous clusters of galaxies (Gorenstein et al. 1978; Branduardi-Raymont et al. 1981; Abramopoulos & Ku 1983; Jones & Forman 1984).

The value of β was first measured from fits of the β-model to the surface brightness distribution of clusters. Using *Einstein* X-ray data for rich clusters, Jones & Forman (1984) found an average value of $\beta_{\rm fit} \simeq 0.64$, while for groups, Kriss, Cioffi, & Canizares (1983) found $\beta_{\rm fit} \simeq 0.41$. These values of $\beta < 1$ indicate that the intracluster medium (ICM) is hotter and more extended than the galaxies. If the gas is in equilibrium with the dark matter, then $\beta < 1$ indicates that the galaxies have a velocity dispersion smaller than the value predicted by the virial theorem.

This measure has been largely debated. In fact, the parameter β can also be evaluated from optically determined galaxy velocity dispersions and direct spectroscopic X-ray temperaures of the gas. The value $\beta_{\rm spec}$ measured in this way is found to be larger than $\beta_{\rm fit}$ by a factor of 1.5–2 (Mushotzky 1984, 1988; Sarazin 1986). Evrard (1990) found the same discrepancy by performing a hydrodynamic simulation of the ICM. This is attributed to ignoring incomplete thermalization of the gas and to assuming a poor modeling of the underlying binding mass distribution.

Edge & Stewart (1991) analyzed a sample of 23 clusters of galaxies, finding an average value of $\beta_{\rm spec} \simeq 0.91$ which is reduced to 0.83 after exclusion of the peculiar clusters Perseus and A2147, which have $\beta_{\rm spec} \simeq 1.8$. The better agreement of this measure of $\beta_{\rm spec}$ with $\beta_{\rm fit}$ is due mainly to the better quality of the optical data (which has reduced the high velocity dispersion in several clusters) rather than to changes in measured temperatures or to a different composition of the samples used to derive $\beta_{\rm spec}$. The overestimated values of the velocity dispersion may be due to the presence of significant velocity substructures as found by Fitchett & Smail (1991) in the Perseus Cluster (a value $\beta_{\rm spec} > 1$ has been suggested as an indicator of substructures in clusters). The conclusion of Edge & Stewart (1991) is that both $\beta_{\rm fit}$ and $\beta_{\rm spec}$ are less than 1. Finally, Gerbal, Durret, & Lachièze-Rey (1994) observe that there is no contradiction between theory ($\beta_{\rm fit} = \beta_{\rm spec}$) and observations ($\beta_{\rm fit} < \beta_{\rm spec}$) and that the discrepancy was only the consequence of an oversimplification of the dynamical problem.

Recently, Lubin & Bahcall (1993) analyzed a sample of 41 clusters of galaxies for which both T and $\langle v^2 \rangle^{1/2}$ were observationally determined, finding an average $\beta_{\rm spec} = 0.94 \pm 0.08$ compatible with a β-value equal to 1. The higher percentage of clusters with line-of-sight galaxy velocity dispersion greater than 1000 km s^{-1} present in this sample ($\sim$30%) with respect to that analyzed by Edge & Stewart ($\sim$17%) may be the origin of the difference between the two measures.

Bahcall & Lubin (1994) discussed the possibility of resolving the β-discrepancy using the observed radial galaxy distribution in rich clusters, $n(r) \sim r^{-2.4\pm0.2}$ for $1\ h_{50}^{-1} \leq r \leq 3\ h_{50}^{-1}$ Mpc (Seldner & Peebles 1977; Peebles 1980), instead of the previously assumed King approximation, $n(r) = n_0(1 + r^2/r_c^2)^{-3/2} \rightarrow r^{-3}$ for r much larger than the core radius r_c.[1] They found a corrected $\beta_{\rm fit}^c \simeq 0.84 \pm 0.07$, which is compatible with the value $\beta_{\rm spec} \simeq 0.94 \pm 0.08$ derived from Lubin & Bahcall (1993).

However, for $r < 1\ h_{50}^{-1}$ Mpc, the King model gives an accurate description of the density profiles. Thus, to provide a good fit, the Seldner & Peebles distribution must be normalized to give the same density as the King model at $r = 1\ h_{50}^{-1}$ Mpc. When this is done, a smaller correction obtains, and $\beta_{\rm fit}^c \leq 0.7$ for a mean value of $\beta_{\rm fit} = 0.64$. In addition, it remains difficult to explain with this argument the differences between groups and rich clusters and, in particular, to make galaxy groups compatible with $\beta \simeq 1$.

We also note that even an isothermal galaxy distribution yields $\beta_{\rm fit} < 1$ for the Coma Cluster (Fusco-Femiano & Hughes 1994). Thus, all the physically motivated galaxy distributions used so far give $\beta_{\rm fit} < 1$, so that the problem seems not to derive from the specific galaxy distribution used to fit the data.

A possible solution might consist in assuming an additional energy source for the gas in the form of galactic winds (see, e.g., the numerical simulation by David, Forman, & Jones 1991 concerning the interstellar medium of elliptical galaxies). However, White (1991) showed that such energy input is negligible for rich clusters.

The alternative which will be explored in this paper, is that $\langle v^2\rangle^{1/2} < \sigma$ (velocity bias). In this case, the gas is actually in equilibrium with the potential so that $\mu m_p \sigma^2/3kT = 1$, but $\beta = \langle v^2\rangle/\sigma^2 = b_v^2 < 1$.

The velocity bias has been found in numerous N-body simulations (Carlberg & Dubinski 1991; Evrard, Summers, & David 1992; Katz & White 1993; Carlberg 1994) and implies an energy loss of the galaxy population. Its value from the simulations is $0.7 \leq b_v \leq 0.9$, in agreement with the X-ray observations. A recent three-dimensional hydrodynamical N-body simulation of a Coma-sized cluster of galaxies (Metzler & Evrard 1994) found a velocity bias $b_v \approx 0.85$, which yields $\beta < 1$.

Recent simulations by Carlberg (1994) show that velocity bias in clusters is a dynamical effect, arising in the absence of a viscous or dissipating gas component. The origin of the effect is still unclear.

In this paper we propose galaxy aggregation as a possible mechanism leading to velocity bias. In fact, during galaxy merging, the orbital energy is transferred to the internal degrees of freedom, thus "cooling" the galaxy velocity distribution despite the counteracting effect of the cluster gravitational potential. In particular, we study the modification of the galaxy velocity distribution $f(\boldsymbol{v})$ due to binary mergings. We start at $t = 0$ with a Maxwell distribution with velocity dispersion $\langle v^2\rangle = \int f(\boldsymbol{v})v^2\, d^3v$ equal to the dark matter σ, so that initially $\beta(t = 0) = \langle v^2\rangle_{t=0}/\sigma^2 = 1$. Then we investigate the possibility that galaxy aggregations will modify $f(\boldsymbol{v})$ at later times, thus producing an asymptotic $\langle v^2\rangle < \sigma^2$ (velocity bias) so that $\beta < 1$.

To follow the time evolution of the velocity distribution, we solve the complete (collisional) Boltzmann-Liouville equation including aggregations, which we introduce in § 2. In § 3 we propose a perturbative approach, used in § 4 to derive numerical solutions.

The evolution of the galaxy distribution results from the opposite action of two effects: the galaxy collisions (tending to shift the velocity distribution toward small velocities) and the galaxy response to the cluster gravitational field (tending to balance the effect of the collisions). We discuss the conditions under which this dynamical process is effective in producing a small velocity bias and the dependence of the results on the dark matter velocity dispersion σ (characterizing the deepness of the cluster potential wells), on the cluster size, and on the shape of the cluster density distribution. Finally, we compare our results with the observed dependence of β on the cluster characteristics (§ 5).

2. COLLISIONAL BOLTZMANN-LIOUVILLE EQUATION FOR AGGREGATIONS

One possible mechanism for a decrease of the galaxy average velocity may be provided by the transfer of orbital energy to the galaxies' internal degrees of freedom, which can be realized through galaxy mergings. To study the evolution of the velocity distribution (and of its moments), we solve the Boltzmann-Liouville equation for the phase-space density $F(\boldsymbol{v}, \boldsymbol{r})$ of colliding galaxies in the cluster gravity field (with potential ψ), which are allowed to aggregate with cross section Σ. This equation reads

$$\frac{\partial F(\boldsymbol{v}, \boldsymbol{r})}{\partial t} + \boldsymbol{v} \cdot \frac{\partial F(\boldsymbol{v}, \boldsymbol{r})}{\partial \boldsymbol{r}} + \frac{\partial \psi}{\partial \boldsymbol{r}} \cdot \frac{\partial F(\boldsymbol{v}, \boldsymbol{r})}{\partial \boldsymbol{v}} = \int d^3v_1\, d^3v_2\, v_{\rm rel}(\boldsymbol{v}_1, \boldsymbol{v}_2)\Sigma(v_{\rm rel})F(\boldsymbol{v}_1, \boldsymbol{r}_2)F(\boldsymbol{v}_1, \boldsymbol{r}_2)\delta(\boldsymbol{v}_1 + \boldsymbol{v}_2 - \boldsymbol{v}/2)$$
$$- F(\boldsymbol{v}, \boldsymbol{r}) \int d^3v_1\, v_{\rm rel}(\boldsymbol{v}, \boldsymbol{v}_1)\Sigma(v_{\rm rel})F(\boldsymbol{v}_1, \boldsymbol{r}_1)\ , \quad (2.1)$$

where $\boldsymbol{r} = (r, \theta, \phi)$ and $\boldsymbol{v} = (v, \Theta, \Phi)$ are the particle position and velocity in spherical coordinates, $v_{\rm rel}(\boldsymbol{v}_1, \boldsymbol{v}_2) \equiv |\boldsymbol{v}_1 - \boldsymbol{v}_2|$ is the modulus of relative velocity of particles 1 and 2, and the δ-function in the first integral represents the conservation of momentum for an aggregation of particles 1 and 2 of equal mass m into a particle with mass $2m$ (all the galaxies are considered to have the same mass). This form of momentum conservation is the one appropriate for aggregation and does not conserve the energy (which is transferred to the internal degrees of freedom not appearing in eq. [2.1]), so that the condition expressed by the δ-function is essential here. It expresses the orbital energy loss of the galaxy population.

The terms involving space and velocity derivatives on the left-hand side of equation (2.1) express the variation of $F(\boldsymbol{v}_1, \boldsymbol{r}_1)$ due to the usual Boltzmann term (including the action of the gravitational potential), while the right-hand side expresses the changes due to aggregations following galaxy collisions. We now assume the space density $n(r, \theta, \phi)$ and the velocity distribution $f(v, \Theta, \Phi)$ to be

[1] Here h_{50} is the Hubble constant normalized to 50 km s^{-1} Mpc.

isotropic and independent, so that

$$F(\boldsymbol{v}, \boldsymbol{r}) \equiv n(r) f(v) . \tag{2.2}$$

In the following we shall also assume that the space density has a self-similar shape, i.e.,

$$n(r) = n_0 \, q(x) \qquad \text{with} \qquad x = r/r_c , \tag{2.3}$$

where n_0 is the central density and r_c is a characteristic radius for the distribution. This ansatz subsumes most of the more commonly used $n(r)$ (see King 1966; Bahcall & Sarazin 1977). In fact, we are going to integrate over $\boldsymbol{r}$, so the detailed shape of the space density is not crucial.

Using the ansatz (2.2) and (2.3) we can integrate equation (2.1) over the position $\boldsymbol{r}$ to obtain [the temporal argument of $f(v)$ will be omitted for simplicity]

$$\alpha \frac{\partial f(v)}{\partial t} + \frac{\eta}{r_c} v f(v) + \left[\int d^3 r \, B(r) \right] \frac{\partial f(v)}{\partial v} = n_0 \, \gamma S(\boldsymbol{v}) , \tag{2.4a}$$

$$S(\boldsymbol{v}) = \int d^3 v_1 \, d^3 v_2 \, v_{\rm rel}(\boldsymbol{v}_1, \boldsymbol{v}_2) \Sigma(v_{\rm rel}) f(v_1) f(v_2) \delta(\boldsymbol{v}_1 + \boldsymbol{v}_2 - \boldsymbol{v}/2) - f(v) \int d^3 v_1 \, v_{\rm rel}(\boldsymbol{v}, \boldsymbol{v}_1) \Sigma(v_{\rm rel}) f(v_1) . \tag{2.4b}$$

Here $\alpha = \int dx \, x^2 q(x)$, $\eta = \int dx \, x^2 \partial_x \, q(x)$, and $\gamma = \int dx \, x^2 q^2(x)$ result from integrating the space density over the position and extend over the entire volume of the cluster. The function $B(r)$ includes the dependence on the potential $\psi(r)$; we will discuss this term later.

Using the constraint on momentum conservation expressed by the δ-function, the first term of $S(\boldsymbol{v})$ in equation (2.4b) reduces to integration only over $\boldsymbol{v}_1$. In this term, after the momentum conservation constraint, $v_2 = |\boldsymbol{v}_2| = |\boldsymbol{v}/2 - \boldsymbol{v}_1|$ and $v_{\rm rel}(\boldsymbol{v}_1, \boldsymbol{v}_2) = |\boldsymbol{v}_1 - \boldsymbol{v}_2| = |2\boldsymbol{v}_1 - (1/2)\boldsymbol{v}|$.

We integrate the resulting equation again over $d\Omega \equiv d \cos \Theta \, d\Phi$ (the angular components of $\boldsymbol{v}$) to get an equation only in the variables t and v, which reads

$$\alpha \frac{\partial f(v)}{\partial t} + \frac{\eta}{r_c} v f(v) + \left[\int d^3 r \, B(r) \right] \frac{\partial f(v)}{\partial v} = n_0 \, \gamma S(v) , \tag{2.5a}$$

$$S(v) = \frac{1}{4\pi} \int d\Omega \left[\int dv_1 \, v_1^2 \, d\Omega_1 \, v_{\rm rel} \, \Sigma(v_{\rm rel}) f(v_1) f(|\boldsymbol{v}/2 - \boldsymbol{v}_1|) - f(v) \int dv_1 \, v_1^2 \, d\Omega_1 \, v_{\rm rel} \, \Sigma(v_{\rm rel}) f(v_1) \right] , \tag{2.5b}$$

where $d\Omega_1 = d \cos \Theta_1 \, d\Phi_1$.

3. PERTURBATIVE APPROACH

To solve equation (2.5) we adopt a perturbative approach. We start at $t = 0$ with no collisions, and we look for solutions of the form

$$f(v, t) = f_0(v) + f_p(v, t) , \tag{3.1}$$

where f_0 is the time-independent initial condition which must satisfy the noncollisional equation [eq. (2.5) with $S(v) = 0$] and $f_p(v, t)$ is a small perturbation due to the onset of collisions. We now impose the condition that the unperturbed solution is a Maxwell distribution with velocity dispersion σ (Lynden-Bell 1967; Shu 1978):

$$f_0(v) = \left(\frac{1}{2\pi\sigma^2} \right)^{3/2} e^{-(1/2)v^2/\sigma^2} . \tag{3.2}$$

On substituting equations (3.1) and (3.2) into equation (2.5a) with $S(v) = 0$, the time derivative is null and we obtain the relationship

$$\int d^3 r \, B(r) = \sigma^2 \eta / r_c , \tag{3.3}$$

so that equation (2.5) is now fully determined and reads

$$\alpha \frac{\partial f_p(v)}{\partial t} + \eta \frac{\sigma}{r_c} \left[\frac{v}{\sigma} f_p(v) + \sigma \frac{\partial f_p(v)}{\partial v} \right] = n_0 \, \gamma S(v) , \tag{3.4a}$$

$$S(v) = 4\pi \left[\int dv_1 \, v_1^2 v_{\rm rel} \, \Sigma f(v_1) f(|\boldsymbol{v}/2 - \boldsymbol{v}_1|) - f(v) \int dv_1 \, v_1^2 v_{\rm rel} \, \Sigma f(v_1) \right]_{\overline{\Omega, \Omega_1}} , \tag{3.4b}$$

where $\overline{\Omega, \Omega_1}$ indicates the average over the solid angles Ω and Ω_1. Note that the effect of the gravitational potential [entering $B(r)$ in eq. (2.5a) is taken account in the left-hand side of equation (3.4a) through the substitution (3.3).

First, we focus on the kernel of the collisional term. We adopt the following expression for the cross section (Saslaw 1985):

$$\Sigma = \pi R_m^2 \left[1 + \frac{G(m_1 + m_2)}{R_m \, v_{\rm rel}^2} \right] \qquad \text{with} \qquad R_m = r_g \left(\frac{v_g}{\sigma} \right)^{1/3} , \tag{3.5}$$

where r_g is the radius of the galaxies (taken to be the same size) and v_g is the velocity dispersion of stars inside the galaxies; R_m is then the distance at closest approach. The cross section is the sum of a geometrical term $\sim \pi r_g^2$ and a term describing the focusing effect of gravity.

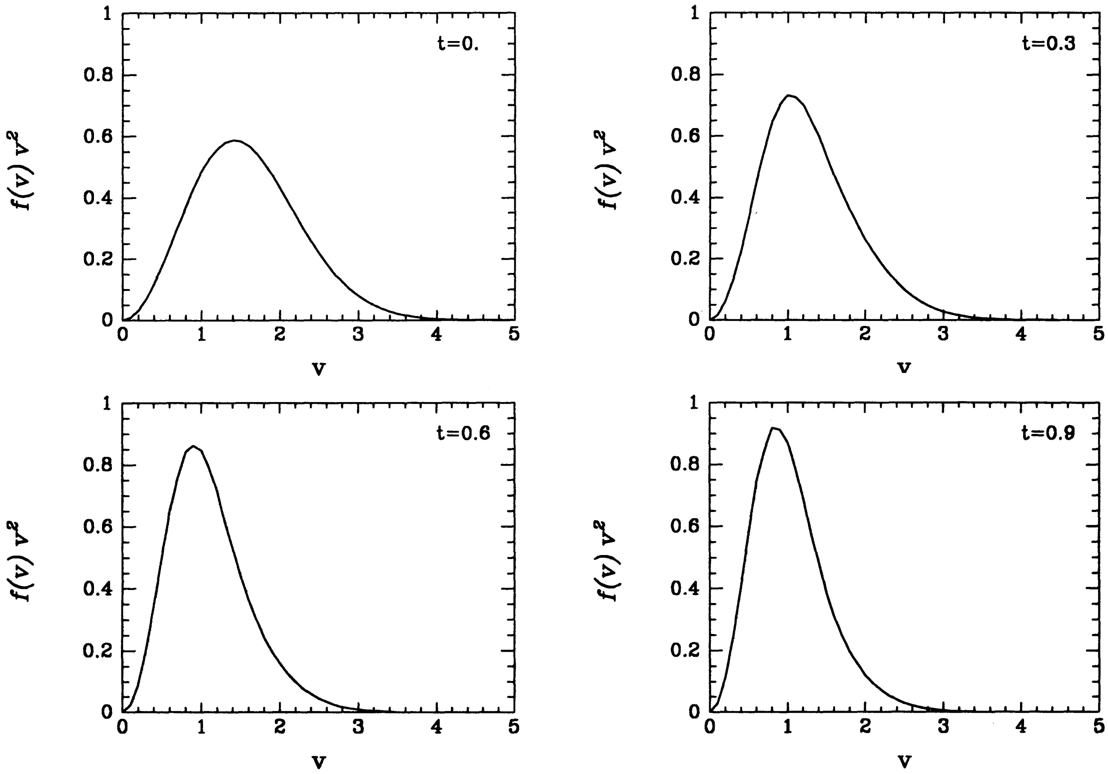

FIG. 1.—Effect of the collision term only [the expression $S(\tilde{v})$ in eq. (3.7)] on the galaxy velocity distribution. For increasing time, the distribution tends to shift toward smaller velocities.

Note that the total number of galaxies in a cluster is

$$N_{\rm tot} = 4\pi n_0 r_c^3 \int dx\, x^2 q(x) = 4\pi\alpha n_0 r_c^3 \; ;$$

the product $n_0 v_{\rm rel} \Sigma$ characterizing the amplitude of the collisional term then takes the following form

$$v_{\rm rel}\, \Sigma(v_{\rm rel}) = \left(\frac{\sigma}{r_c}\right) A(\sigma) K(\tilde{v}_{\rm rel}) \,, \tag{3.6a}$$

$$A(\sigma) = \frac{N_{\rm tot}}{4\alpha} \left(\frac{r_g}{r_c}\right)^2 \left(\frac{v_g}{\sigma}\right)^{2/3} , \tag{3.6b}$$

$$K(\tilde{v}_{\rm rel}) = \tilde{v}_{\rm rel}\left[1 + \frac{1}{\tilde{v}_{\rm rel}} \left(\frac{v_g}{\sigma}\right)^{5/3}\right] , \tag{3.6c}$$

where $\tilde{v}_{\rm rel} = v_{\rm rel}/\sigma$. Note that the collisional term is now expressed as a product of the dimensional factor σ/r_c (the same appearing in the noncollisional term in eq. [3.4]) times an adimensional amplitude (not depending on velocity) times an adimensional kernel K which contains the dependence on velocities through the adimensional quantity $\tilde{v}_{\rm rel} \equiv v_{\rm rel}/\sigma$.

Thus, we can finally write equation (3.4) in a compact and *adimensional* form:

$$\frac{\partial f_p(\tilde{v})}{\partial \tau} = \gamma A(\sigma) S(\tilde{v}) - \eta Q(\tilde{v}) \,, \tag{3.7a}$$

$$Q(\tilde{v}) = \tilde{v} f_p(\tilde{v}) + \frac{\partial f_p(\tilde{v})}{\partial \tilde{v}} \,, \tag{3.7b}$$

$$S(\tilde{v}) = 4\pi \left[\int d\tilde{v}_1\, \tilde{v}_1^2 K(\tilde{v}_1, \tilde{v}_2) f(\tilde{v}_1) f(\tilde{v}_2) - f(\tilde{v}) \int d\tilde{v}_1\, \tilde{v}_1^2 f(\tilde{v}_1) K(\tilde{v}, \tilde{v}_1) \right]_{\overline{\Omega,\Omega_1}} , \tag{3.7c}$$

where $\tau \equiv t(\sigma/\alpha r_c)$ is an adimensional time and $\tilde{v} = v/\sigma$, $\tilde{v}_1 = v_1/\sigma$, and $\tilde{v}_2 = |\boldsymbol{v}/2 - \boldsymbol{v}_1|/\sigma$ are adimensional velocities. Let us discuss equation (3.7a). The changes in the velocity distribution result from a balance of two terms: the collisions, expressed by $\gamma A(\sigma) S(\tilde{v})$, and the response to the gravitational field, expressed by $\eta Q(\tilde{v})$.

The collisions tend to modify the galaxy velocity distribution, stretching the initial Maxwell distribution toward smaller velocities (see Fig. 1), thus giving a smaller β. The effectiveness of such stretching depends only on $\gamma A(\sigma)$. $A(\sigma)$ depends on the environment in

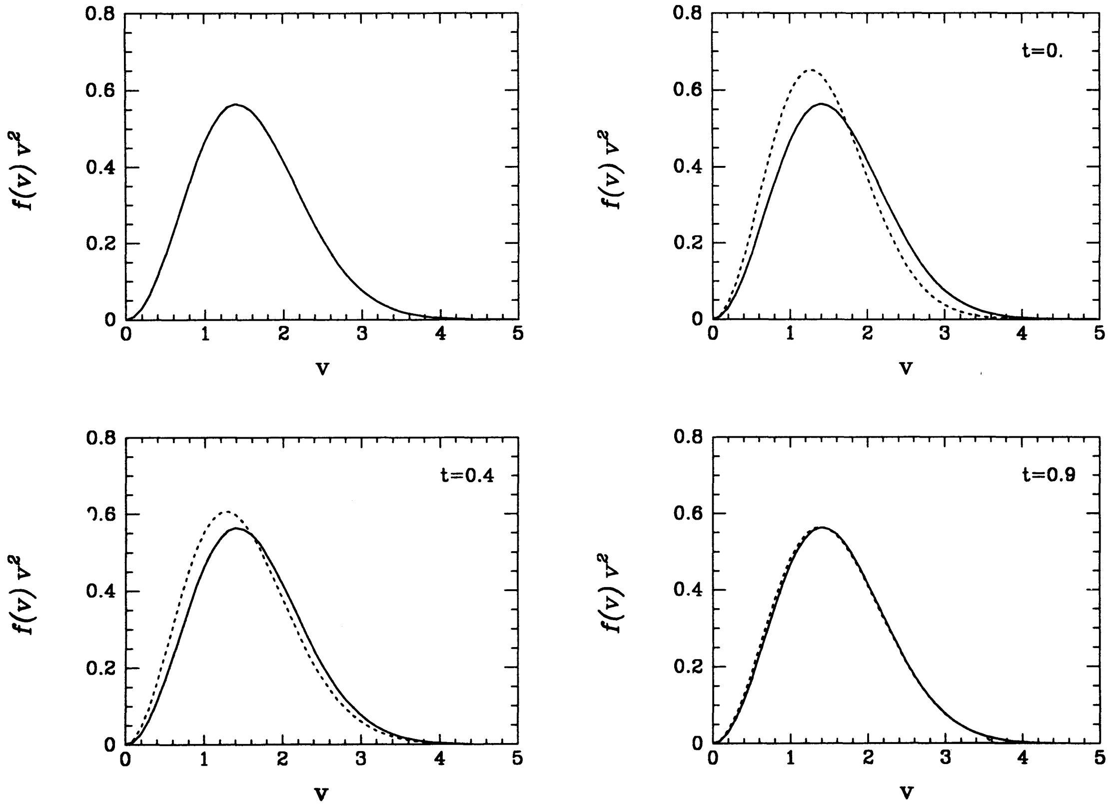

FIG. 2.—Illustration of the effect of the Boltzmann term $Q(\tilde{v})$ (see eq. [3.7]) representing the effect of the cluster potential. The initial Maxwell distribution (*solid line in all panels*) is perturbed at $t = 0$ (*dotted line in top right panel*). The velocity distributions at subsequent times (*dotted lines*) show that the system in absence of collisions tends to return to the equilibrium Maxwell distribution.

which merging takes place: the smaller σ (i.e., the poorer the galaxy cluster), the larger the effect. Also, decreasing the cluster size (i.e., r_c) gives larger $A(\sigma)$ (see eq. [3.6b]).

The stretching of $f(v)$ due to aggregations is counteracted by the term $Q(\tilde{v})$ of equation (3.7). This, at each time step $d\tau$, tends to bring $f(v)$ back to the equilibrium Maxwell distribution (see Fig. 2), due to the effect of the gravitational potential.

The delicate balancing between the two terms is governed by the coefficients α, η, and γ [derived from the integration of the spatial distribution $q(x)$ of galaxies in the cluster; see eqs. (2.3) and (2.4) and below] and by the collisional amplitude $A(\sigma)$ which depends on the galactic and cluster parameters.

The timescale of this process is given by $\alpha r_c/\sigma$, which defines the adimensional time τ. For a King distribution, r_c is of the order of one-eighth the cluster virial radius R_v and $\alpha \sim 1$, so that τ is the time in units of approximately $t_d/8$, where $t_d = 1/(2\pi G\rho)^{1/2} \sim R_v/\sigma$ is the cluster dynamical time (ρ is the average cluster density).

4. NUMERICAL SOLUTIONS

We solve the adimensional integrodifferential equation (3.7) by numerical integration. The procedure is the following: We start at $\tau = \tau_0$ with $f_p(v, \tau_0) = 0$ (eq. [3.1]), so that initially $Q(\tilde{v}) = 0$, and solve equation (3.7) for $f_p(v, \tau_0 + \Delta\tau) = [\gamma A(\sigma)S(\tilde{v}) - \eta Q(\tilde{v})]\Delta\tau$. Then we substitute into equation (3.1) to get $f(v, \tau_0 + \Delta\tau)$. These functions are inserted back into the complete equation (3.7), and the whole procedure is iterated.

The averages over the angular coordinates Ω and Ω_1 (which enter in the computation of $\tilde{v}_{rel}$ and $\tilde{v}_2$ from any given $\tilde{v}$ and $\tilde{v}_1$) are performed by Monte Carlo integration with $\sim 10^5$ extractions. The integral over $\tilde{v}_1$ is calculated over discrete sums with a step $\Delta\tilde{v}_1 \simeq 10^{-1}$. The time integration is performed with a step $\Delta\tau = 10^{-1}$ ($\approx 1/100$ of the dynamical time t_d).

To evaluate the coefficients α, η, and γ, we use a King profile for the space density $q(x) = (1 + x^2)^{-3/2}$. The detailed shape of $q(x)$ is not crucial because only integrals of $q(x)$ are important here. We obtain $\alpha = 1.125$, $\eta = -1.3$, and $\gamma = 0.2$ when the integrations over x are performed up to $x_{max} = r_{max}/r_c = 4$, corresponding to a cluster region where the isothermal approximation is valid.

The collisional amplitude $A(\sigma)$ depends on the galactic and cluster parameters. The former are set to the typical values $r_g = 50$ kpc and $v_g = 250$ km s^{-1}.

The dependence of the amplitude of the collisional term on the cluster parameters is $A(\sigma) \propto N_{tot} r_c^{-2} \sigma^{-2/3}$. Observed correlations give $N_{tot} \propto \sigma^{2.2}$ (Bahcall 1981) and $r_c^2 \propto \sigma^2$, so that the product $N_{tot} r_c^{-2}$ is *on average* almost a constant. Thus, in the following we will use $N_{tot} = 10^3$ (for the Coma Cluster, Godwin & Peach 1977 measured magnitudes of 923 galaxies) and $r_c = 250$ kpc (Bahcall 1975) and keep only σ as a variable describing the cluster. However, for a given σ, a scatter will exist around the results. We expect clusters with large galaxy density and small core radius to have larger collisional amplitudes and hence smaller values of β.

With the above coefficients determined, we solve equation (3.7). In Figure 3, we show the evolution of the velocity distribution with time, for a Coma-like cluster with one-dimensional dark matter velocity dispersion $\sigma_r = \sigma/(3)^{1/2} = 1100$ km s^{-1} (corresponding to a gas temperature $kT = \mu m_p \sigma_r^2 \approx 8$ keV characteristic of the Coma Cluster; Hughes & Tanaka 1992). The collisional term, tending to stretch the distribution toward small velocities, is not completely balanced by the phase-space diffusion [the term $Q(\tilde{v})$ in eq. (3.7)]. As a net result, the average square velocity shifts to smaller values. From the velocity distribution we compute β from the relationship

$$\beta = \frac{\langle v^2 \rangle}{\sigma^2} = \langle \tilde{v}^2 \rangle = 4\pi \int f(\tilde{v})\tilde{v}^4 \, d\tilde{v} \tag{4.1}$$

with the normalization $\langle \tilde{v} \rangle_{t=0} = 1$.

In Figure 4, the resulting β is plotted as a function of time. After $\approx 1.5\ t_d$, it tends to an asymptotic value of ≈ 0.65, which characterizes a final stable state.

The dependence of the asymptotic β on the cluster velocity dispersion is shown in Figure 5 and is inverse, as expected. Thus, the observed anticorrelation between β and σ is naturally explained in this model. The three curves in Figure 5 refer to different combinations of cluster and galactic parameters to be inserted in equation (3.6b). Note that, for example, increasing the cluster size

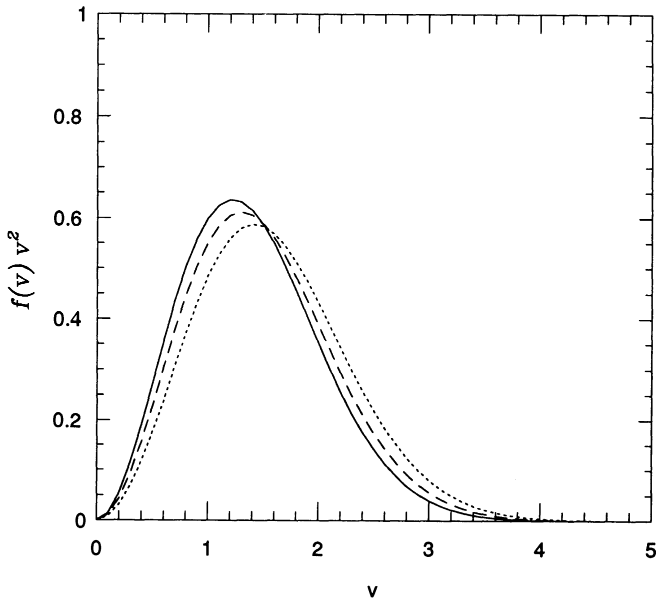

FIG. 3.—Time evolution of the galaxy velocity distribution following the complete eq. (3.7). The dotted line corresponds to $t = 0$, dashed line to $t = 0.8\ t_d$, and solid line to $t = 1.5\ t_d$.

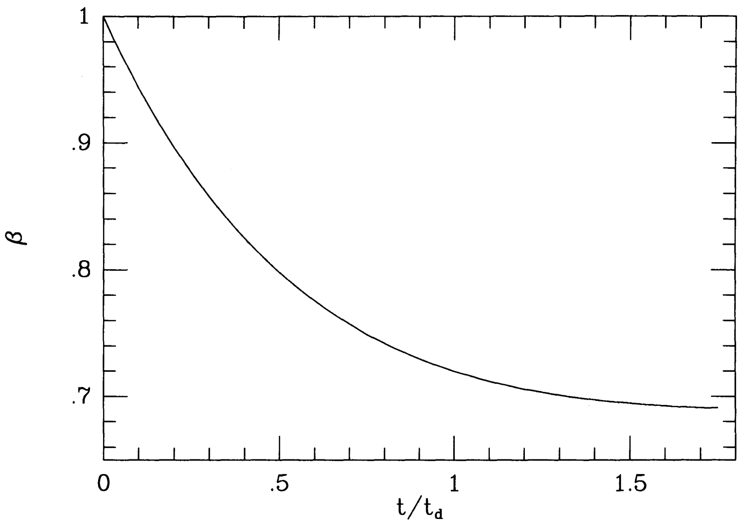

FIG. 4.—Ratio $\langle v^2 \rangle/\sigma^2 = \beta$ as a function of time. The parameters used in the computation are given in the text. The dark matter one-dimensional velocity dispersion is $\sigma_r = \sigma/\sqrt{3} = 1100$ km s^{-1}. Note the asymptote for $t > 1.5 t_d$.

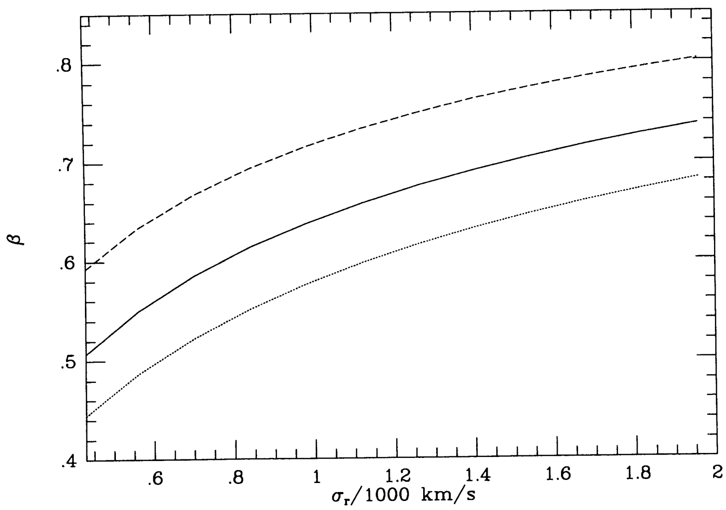

FIG. 5.—Dependency of β on the dark matter one-dimensional velocity dispersion $\sigma_r = \sigma/\sqrt{3}$ for three different combinations of galaxy and cluster parameters; see eq. (3.6b). The solid line corresponds to $N_{\rm tot} = 1000$, $r_g = 50\ h_{50}^{-1}$ kpc, $v_g = 250$ km s^{-1}, and $r_c = 250\ h_{50}^{-1}$ kpc, which yield $A(\sigma) = 3.5(\sigma/1000\ {\rm km\ s^{-1}})^{-2/3}$. The other curves correspond to changes in the parameters so as to give $A(\sigma) = 2.5(\sigma/1000\ {\rm km\ s^{-1}})^{-2/3}$ (*dashed line*) and $A(\sigma) = 4.5(\sigma/1000\ {\rm km\ s^{-1}})^{-2/3}$ (*dotted line*).

r_c decreases the collisional amplitude (according to eq. [3.6b]) and yields a larger value of β, as expected. A similar correlation of β with the core radius r_c is actually observed (see Jones & Forman 1984).

5. DISCUSSION AND CONCLUSIONS

We studied the effect of galaxy aggregations in clusters on the velocity distribution. Starting from an initial Maxwell distribution characterized by a galaxy velocity dispersion equal to that of the dark matter, we solved the complete Boltzmann-Liouville equation, finding an asymptotic final distribution characterized by a smaller galaxy velocity dispersion. This form of velocity cooling is due to orbital energy transfer to internal energy occurring in galaxy merging, not completely balanced by the response to the cluster gravitational potential. If the intracluster gas is in equilibrium with the cluster potential, then our results imply that β (the ratio of the energy per unit mass in galaxies to that in gas) is 0.5–0.7.

The computation we present is based on the following assumptions:

1. The cluster galaxies are all assumed to have the same size, 50 kpc, and internal velocity dispersion, $v_g = 250$ km s^{-1}. The effect of the inclusion of a nonuniform mass distribution has been evaluated by Monte Carlo simulations of aggregation processes in the simplified case of constant cluster potential. A very small dependence of the resulting galaxy velocity distribution on the initial galaxy mass function has been found.
2. The initial condition from which we compute the effect of aggregations is taken to be the relaxed, virialized state following violent relaxation (Lynden-Bell 1967). Hence, the present computation does not apply to clusters showing evident substructures, such as Perseus and A2147, which in fact have an observed $\beta > 1$.
3. The galaxy velocity distribution is assumed isotropic and independent of position, and the galaxy space distribution is assumed to be isotropic at all times. These assumptions are realistic because aggregation processes do not sensibly distort the spatial distribution starting from the initial conditions described above (Menci, Colafrancesco, & Biferale 1993).

Our results are robust with respect to the galaxy density profile $q(x)$ used in the computation. In fact, it enters into the computation only in integral forms (through the coefficients α, η, and γ), which are not very sensitive to the details of the shape of $q(x)$.

The dependencies shown in Figures 3, 4, and 5 are in good agreement with existing observations. The existence of an asymptote in the time evolution of β (Fig. 4) indicates the existence of a final *stable* state determined by the complete dynamics. This state is characterized by a velocity distribution whose shape is indistinguishable from a Maxwellian (see Fig. 3) but is characterized by a galaxy velocity dispersion $\langle v^2 \rangle^{1/2}$ smaller than the dark matter σ.

The asymptotic value of $\langle v^2 \rangle^{1/2}$ is in good agreement with observed β. The curve shown in Figure 4 is obtained using the parameters characteristic of Coma-like clusters: $N_{\rm tot} = 10^3$, $r_c = 250\ h_{50}^{-1}$ kpc, and σ_r (the one-dimensional dark matter velocity dispersion) equal to 1100 km s^{-1}. We obtain an asymptotic $\beta \approx 0.65$ in good agreement with the accurately observed value $\beta_{\rm fit} = 0.64^{+0.03}_{-0.05}$ (Fusco-Femiano & Hughes 1994) for the Coma Cluster. This is also in rough agreement with the value $\beta_{\rm spec} = 0.77^{+0.11}_{-0.09}$ (Edge & Stewart 1991). However, for very unrelaxed clusters, the observed $\beta_{\rm spec}$ is particularly affected by substructures and inhomogeneities, so that the comparison of $\beta_{\rm spec}$ with our results (obtained under the assumption of cluster relaxation and isotropy) is critical.

Also, the predicted dependence of β on the cluster dark matter velocity dispersion σ (see Fig. 5) is consistent with the decrease of the average β with σ, from the value of 0.64 for rich clusters (Jones & Forman 1984) to 0.41 for groups (Kriss et al. 1983) determined from fits to the surface brightness data. A similar tendency has also been observed from spectroscopic measures of β (Edge & Stewart 1991). Our model also predicts a correlation of r_c with β (see Fig. 5), which is actually observed (Jones & Forman 1984).

The value of velocity bias we find is consistent with $b_v \approx 0.7$–0.8 found in clusters from N-body simulations (Carlberg & Dubinski 1991; Evrard et al. 1992; Katz & White 1993; Carlberg 1994; Metzler & Evrard 1994), which constitute so far the only *direct* evidence for a velocity bias, apart from the X-ray observations.

One implication of $b_v < 1$ is that estimating the mass of clusters of galaxies from the galaxy velocity dispersions may lead to an underestimate of the cluster mass, a possibility that can be tested by weak gravitational lensing techniques (see Fahlman et al. 1995). This effect, in turn, could imply that the value of the density parameter Ω_0 measured from cluster virial masses is underestimated (Carlberg 1994).

A final implication specific to our model is that, if the energy is transferred from the galaxy orbital motion to internal degrees of freedom, star formation should be triggered by merging (see Lacey & Silk 1991; Broadhurst, Ellis, & Glazebrook 1992) and galaxies should become bluer as the merging activity increases. But the latter effect is larger for denser clusters, which in turn form at earlier epochs, so that a blueing of galaxies with redshift should be found (Cavaliere & Menci 1993). This is actually observed (Butcher & Oemler 1984). Thus, velocity bias, $\beta < 1$, and the Butcher-Oemler effect could be different observational aspects of a unique process.

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