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AGN FEEDBACK ON THE INTRACLUSTER MEDIUM

A. CAVALIERE AND A. LAPI
Astrofisica, Dipartimento di Fisica, Universitá 'Tor Vergata', Roma, I-00133
AND
N. MENCI
Osservatorio Astronomico di Roma, Monteporzio Catone, I-00044
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ABSTRACT

Galaxy groups are quite underluminous in X-rays compared to clusters, so the intracluster medium has to be considerably underdense in the former. We consider this is due to substantial energy fed back into the ICM when the baryons in the member galaxies condense into stars ending up in SNe, or accrete on to central supermassive black holes energizing AGNs. We compute the outflow and the blowout effects set by the AGNs and the resulting, steep luminosity-temperature correlation $L_X - T$. We compare this with the SN contribution and with the X-ray data; the latter require the AGN energy to be coupled to the surrounding ICM at fractional levels around $5\,10^{-2}$. We link such $L_X - T$ behavior with the parallel effects of the AGN feedback on the gas in the host galaxy; we find that these yield a correlation steep up to $M_\bullet \propto \sigma^5$ between the galactic velocity dispersions and the central BH masses.

Subject headings: galaxies: clusters: general - hydrodynamics - intergalactic medium - quasars: general - X-rays: galaxies: clusters

1. INTRODUCTION

Groups and clusters of galaxies shine in X-rays due to the thermal bremsstrahlung emission by the hot intracluster medium (ICM) they contain.

But the poorer groups are found to be progressively underluminous, so as to lie substantially below the simple scaling $L_X = L_{grav} \propto T_v^2$. For the latter to hold, in the luminosity $L_X \propto n^2 R^3 T^{1/2}$ the ICM number density n would have to be proportional to the gravitationally dominant DM mass density ρ, with the ratio $n \sim 10^{-1}\,\rho/m_p$. This constraint adds to the temperature T being close the virial value T_v and the size R scaling as the virial radius $R_v \propto T_v^{1/2}\,\rho^{-1/2}$.

However, the observed $L_X - T$ correlation has a shape more like $L_X \propto T_v^3$ for richness 1 clusters, and in moving toward poor groups it bends further down to $L_X \propto T_v^5$ or steeper (Kaiser 1991; Ponman, Cannon, & Navarro 1999; see also § 3). So in groups the ICM is quite underdense relative to the scaling $n \propto \rho$.

Correspondingly, the central ICM entropy $S/k = \ln kT/n^{2/3}$ deviates upward from the simple scaling $S_{grav} \propto \ln kT_v$, to attain the 'floor' value $e^S \propto kT/n^{2/3} \approx 140$ keV cm^2 in poor groups (Lloyd-Davies, Ponman & Cannon 2000). Deficit L_X and excess S are related model-independently by

$$e^{S-S_{grav}} = \left(\frac{L_X}{L_{grav}}\right)^{-1/3} \left(\frac{T}{T_v}\right)^{7/6}, \qquad (1)$$

whenever the ICM is in hydrostatic equilibrium in a DM potential well with depth marked by kT_v.

The high entropy floor indicates that the density deficit in groups occurs in association with an increase of T/T_v. Such a strongly non-adiabatic behavior may be traced back to energy added to the ICM in equilibrium, when the baryons in the member galaxies condense into stars followed by SNe, or accrete onto a central supermassive black hole (BH) kindling an AGN.

However, two issues stand in the way. Not only any effective energy discharge by sources has to compete with the huge equilibrium thermal value $E \approx 10^{61}\,(kT_v/\mathrm{keV})^{5/2}$ erg, but also the degree of its coupling to the ICM is still unknown. Here we compute and discuss two observables that bound or probe the values of such a key parameter at group and at galactic scales.

2. FEEDBACK FROM SNE

Obvious first candidates for energy discharges into the ICM are the SN explosions following star formation in the member galaxies of groups and clusters. Prompt, type II SNe canonically release 10^{51} ergs; these are effectively coupled to the gas when cooperative SN remnants propagate over galactic scales to drive galactic winds (Ostriker & McKee 1988; Wang et al. 2001; Pettini et al. 2001; Heckman 2002). With a coupling around 1/2, the energy input (including winds from hot stars) comes to $\Delta E \approx 3\,10^{48}$ erg$/M_\odot$ per unit solar mass condensed into stars. This would raise by $k\Delta T \approx 0.3$ keV the temperature of the entire ICM in a fiducial group with virial mass $M_v \approx 5\,10^{13}\,M_\odot$ or $kT_v \approx 1$ keV. The outcome looks like a modest $\Delta E/E = \Delta T/T_v \lesssim 1/3$.

Actually, SNe make optimal use of their energy in that they produce *hierarchical preheating* of the ICM; this acts while a group and its ICM are built up hierarchically through merging events with a range of partners. In the process, about half the final DM mass in the main progenitor (and half the ICM mass likewise) is contributed by smaller partners with masses M'_v within the window $M_v/3$ to $M_v/20$, corresponding to T'_v from 0.6 down to $0.15\,T_v$ (Lacey & Cole 1993; Menci & Cavaliere 2000).

The smaller lumps in the window have shallower potential wells and produce more star-related energy on scales closer to their dynamical time t_d; so they are more effective in heating/ejecting their gas share (see also De Zotti 2001). During each subsequent step of the merging hierarchy such gas preheated *externally* to the main progenitor will be less ready to flow into its potential well (see also Muanwong et al. 2002). Thus the effects propagate up the hierarchy, and lower ICM densities will be induced in all structures up to poor clusters.

Many specific models base on hydrostatic equilibrium. Then, given the normalized, scale-invariant potential difference $\Delta\phi$ set by the DM inward of R_v, the ICM density follows $n(r) = n_2 \exp[\beta\,\Delta\phi(r)]$ in the simple isothermal case. This depends on two parameters: $\beta = T_v/T$ (Cavaliere & Fusco-Femiano 1976; Jones & Forman 1984), and the 'boundary' value n_2. Correspondingly, two suppression factors arise in moving from clusters to groups.

One is the outflow effect related to β; this is lowered by about 0.6 from rich clusters toward poor groups where stellar preheating provides a contribution to T comparable to T_v. A second factor is the differential containment expressed by the boundary value n_2. If this is set by jump conditions across the accretion shocks at $r \approx R_v$ (see Takizawa & Mineshige 1998; Gheller, Pantano & Moscardini 1998), the density is further suppressed from clusters to groups by an average factor saturating to $1/2$ (Menci & Cavaliere 2000).

These authors specifically model the process on grafting the ICM hydrostatic equilibrium onto the semi-analytic treatment (SAM) of star and galaxy formation. The latter is based on the hierarchical merging histories of the DM, and includes star formation and gas heating/ejection by SNe in terms of simple phenomenological recipes. These imply heating dominate over ejection at the scales of bright galaxies and larger; gas fractions exceeding some 10^{-1} are only blown out of small galaxies (see also Madau, Ferrara & Rees 2001).

With SN feedback, the SAMs produce good fits to the stellar observables, but arguable agreement (see Borgani et al. 2001) with the $L_X - T$ data at group scales when the Navarro, Frenk & White (1997) potential $\Delta\phi$ and the standard Λ-cosmology are adopted. The resulting $L_X - T$ relation is illustrated in the next Section.

3. FEEDBACK FROM AGNS

The other natural sources of feedback are the AGNs, energized by accretion of cool gas onto supermassive BHs in galactic cores (see Wu, Fabian & Nulsen 2000; Bower et al. 2001). The expected outputs are large, of order $2\,10^{62}\,M_\bullet/10^9\,M_\odot$ ergs for an accreted mass $M_\bullet$, with the standard mass-energy conversion efficiency of order 10^{-1}. If a fraction f is coupled to the surrounding medium, the energy actually injected comes to $\Delta E \approx f\,10^{50}$ erg$/M_\odot$ per unit solar mass condensed into stars; we have used $2\,10^{-3}$ for the ratio of the BH mass to that of the current host bulges (Ferrarese & Merritt 2000; Gebhardt et al. 2000; see also Fabian 2001).

Compared with SNe, AGNs potentially provide a larger energy output in shorter times, close to t_d of the host galaxies. However, f is even more uncertain than the analogous quantity for SNe, though it is conceivably lower. In the 90% radio-quiet AGNs a small coupling $f \approx 10^{-2}$ is expected in view of the low photon momentum and of the flat spectra. Higher values $f \sim 10^{-1}$ are conceivable in systems where the photons are heavily scattered/absorbed within the gravitational reach of the BH, and may escape only in hard X-rays if at all (Fabian, Wilman & Crawford 2002). The 10% radio-loud AGNs have a larger output in kinetic energy of jets, but the evidence to now has shown limited ongoing impacts on the gas surrounding a number of active sources (see McNamara et al. 2001; Terashima & Wilson 2001; Young, Wilson & Mundell 2002).

Whence the interest in probing f from longer term, larger scale effects on the ICM. During the AGN activity the gas initially contained in a group or in a large galaxy will be heated up and partially blown out. We are particularly interested in blowout and outflow *internally* driven in poor groups with $kT_v \sim 1$ keV where $\Delta E/E \sim 1$.

Within the structure's dynamical time t_d we describe the transient regime as a blastwave sweeping through the surrounding gas (see Platania et al. 2002); when $\Delta E/E \lesssim 1$ holds the terminating shock at $r = R_s$ is not necessarily strong, and DM gravity is important.

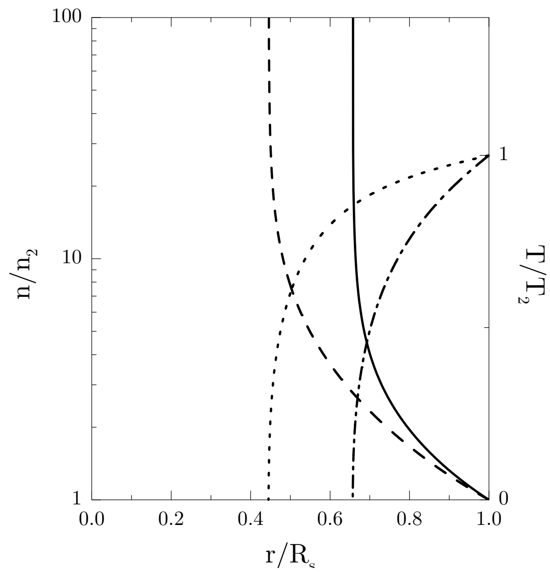

FIG. 1.— Radial distributions of density and temperature (normalized to their postshock values) in a blastwave driven by a flaring AGN. Dot-dashed and dotted lines refer to the temperature behind a strong ($\Delta E/E = 1.8$) or a weak shock ($\Delta E/E = 0.3$), respectively; solid and dashed lines refer to the density. On approaching the piston ($r = 0.66 R_s$ or $r = 0.45 R_s$ for the adopted values of $\Delta E/E$) the density diverges weakly while the mass inside r vanishes.

From the relevant hydrodynamical equations we have derived (and illustrate in fig. 1) a family of self-similar solutions of the Sedov (1959) type that include the DM gravity, a steep initial density gradient, and centrally injected energy $\Delta E(t)$ growing over times of order t_d. The radiative cooling is slow on mass-average in our groups.

Our fiducial case will have $\Delta E(t) \propto t$ injected into an initial configuration with $n(r) \propto r^{-2}$, i.e., isothermal ICM in hydrostatic equilibrium ($\beta \approx 1$) in the potential provided by DM density $\rho(r) \propto r^{-2}$; we denote by $E(R_s)$ the modulus of the total initial energy within the shock radius R_s. In such conditions the terminating shock moves outward at a costant speed v_s, only moderately supersonic when $\Delta E/E \lesssim 1$.

Self-similarity implies $\Delta E(t)/E(R_s)$ to be independent of time and position, as is especially simple to see in our fiducial case where $E(R_s) \propto R_s \propto t \propto \Delta E(t)$. For two values of $\Delta E/E$ we show in fig. 1 the density and temperature runs. The flow begins at a 'piston', the inner contact surface where the density diverges weakly while the gas mass within r and the temperature $T(r)$ vanish.

In fact, the perturbed gas is confined to a shell with outer (shock) radius R_s and inner (piston) radius λR_s. Self-similarity implies the thickness $\Delta R_s/R_s = 1 - \lambda$ of such a shell to depend only on $\Delta E/E$; for strong shocks driven by $\Delta E/E \gg 1$ we find

$\lambda \to 0.84$, while for a weak shock corresponding to $\Delta E/E = 0.3$ we find $\lambda = 0.45$.

These considerations lead us to represent our solutions in terms of the simple 'shell approximation', known to provide results reliable to better than 15%, see Cavaliere & Messina 1976; Ostriker & McKee 1988. In this approximation the energy balance reads

$$\Delta E + E = \frac{1}{2}\, m v_2^2 + \frac{3}{2}\, \bar{p} V - \frac{G M m}{R_s}\ , \qquad (2)$$

and shows the relevance of $\Delta E/E$. Here M is the DM mass within R_s; $V = 4\pi R_s^3 (1-\lambda^3)/3$ is the volume of the shell; m and $\bar{p}$ are the associated gas mass and mean pressure; finally, $v_2 \propto v_s$ is the postshock velocity given by the Rankine-Hugoniot conditions.

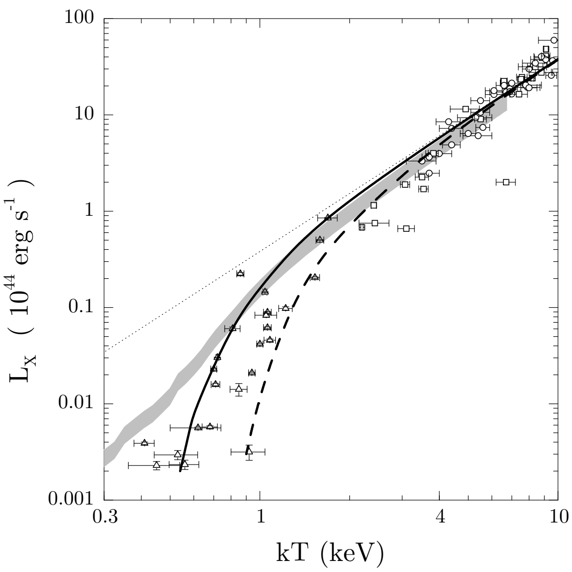

FIG. 2.— The $L_X - T$ correlation; bolometric L_X including standard line emissions. Thin dotted line: gravitational scaling $L_{grav} \propto T_v^2$. The shaded one-sigma strip results from a SAM including the stochastic merging histories of the DM and the SN feedback, see § 2. Thick lines: our results with feedback from AGNs accreting as specified at the end of § 3, and with $f = 3\,10^{-2}$ (solid) and $f = 10^{-1}$ (dashed). Data: Markevitch (1998, circles), Arnaud & Evrard (1999, squares), Helsdon & Ponman (2000, triangles).

Self-similarity requires all terms in eq. (2) to scale like R_s; the coefficients depend only on $\Delta E/E$, and are easily derived following the pattern indicated by the above authors. We find the ratio of the kinetic to the thermal energy (i.e., the 1st to the 2nd term on the r.h.s. of eq. 2) to range from $5\,10^{-2}$ up to 2 when $\Delta E/E$ increases from 0.3 to values larger than 1. Analogously, one may derive the dependence of v_s on $\Delta E/E$, and even an analytic approximation to the mass distribution within the shell.

After the passage of the blastwave and before a major merging event reshuffles the DM mass substantially, the gas recovers hydrostatic equilibrium, and again $n(r) = n_2 \exp[\beta\,\Delta\phi]$ holds; but now the governing parameters are those given in Table 1. The value of $\beta = T_v/\bar{T}$ (related to outflow) is reset using the mass-averaged temperature $\bar{T}$. The new ICM mass $m - \Delta m$ (left over by the blowout) is that still residing at $t = t_d$ between the piston and R_v; then the boundary condition n_2 is reset by requiring consistency with the volume integral of $n(r)$.

We then compute $L_X \propto \bar{T}^{1/2} \int dr\, r^2 n^2(r)$ and plot it as a function of temperature in fig. 2; we approximate $\bar{T}$ with T_v, since these differ only modestly as is seen from the values of β in Table 1. Our results are given for two values of the energy coupled; these bracket $\Delta E = 2\,10^{60}$ erg corresponding, e.g., to $f \approx 5\,10^{-2}$ and to $M_\bullet = 10^9\,M_\odot$ for the largest BH (or sum of BHs) formed in a dynamical time within groups with membership around 10 bright galaxies.

The variable T_v is related to the quantity $\Delta E/E$ that governs the blastwave. On using $\Delta E \propto M_\bullet$ (see the beginning of this Sect.) and $E \propto (kT_v)^{5/2}$ (see § 1), the correspondence is simply given by

$$\frac{\Delta E}{E} = 0.1 \left(\frac{f}{10^{-2}}\right) \left(\frac{M_\bullet}{10^9\,M_\odot}\right) \left(\frac{kT_v}{\mathrm{keV}}\right)^{-5/2} . \qquad (3)$$

In fig. 2 we have actually implemented the second approximation that obtains on inserting on the r.h.s. the factor $[1 + (kT_v/\mathrm{keV})^{6/7}]/2$, slowly varying around 1 keV. This accounts for the cosmological evolution of the AGNs in luminosity and number (see Cavaliere & Vittorini 2002 and references therein) that occurs for redshifts $z < 2$ while groups and clusters with increasing kT_v are built up by the standard hierarchical clustering.

In fig. 2 we also recall the contribution from SNe and report the data.

TABLE 1
PARAMETERS OF THE RECOVERED EQUILIBRIUM

$\Delta E/E$	β	$1 - \Delta m/m^\star$
0.3	0.94	0.92
0.5	0.92	0.82
1	0.86	0.58
1.8	0.8	0.16

* The approximation $\Delta m/m \approx 0.5\,\Delta E/E$ holds to better than 10% for $\Delta E/E < 1.4$.

4. DISCUSSION AND CONCLUSIONS

Two key features are apparent in fig. 2 and are spelled out in Table 1. First, into the cluster range the deviations from the gravitational scaling vanish because both $1 - \Delta m/m$ and $\beta = T_v/\bar{T}$ saturate to 1. Second, moving into the group range the luminosity is non-linearly suppressed as $L_X \propto n_2^2 \propto (1 - \Delta m/m)^2$, due to the increasing contribution from the blowout as $\Delta E/E$ raises toward 1.

The current X-ray data in groups are seen to require values around $f \approx 5\,10^{-2}$; with these, the feedback from AGNs dominates over SNe, causing stronger suppression of L_X and further bending down of the $L_X - T$ relation. Variance of f from $3\,10^{-2}$ to 10^{-1} produces a widening strip, but one still consistent with the current data and their scatter.

As our L_X vs. T intrinsically steepens towards poor groups, we can check it in the adjoining galactic range where cooling still does not dominate. In terms of the velocity dispersion $\sigma = (kT_v/0.6\,m_p)^{1/2}$ from the virial relation, we find a steepening correlation $L_X \propto \sigma^n$; for the upper values of f this has the minimal slope $n \approx 8.5$ in large galaxies with $\sigma = 300$ km s^{-1}, in accord with the detections and the fit by Mahdavi & Geller (2001).

Our specific result is due to a blastwave with $\Delta E/E \approx 1.2$ causing $\Delta m/m \approx 0.6$. Down to what scales can we extend the increasing trend of $\Delta E/E$? We argue $\Delta E/E$ *on average* will not exceed 1 by much, nor will $\Delta m/m$ attain 1.

First, we may just impose the limiting constraint $\Delta E/E \approx 1$ to eq. (3), to find the accreted BH masses

$$M_\bullet \approx 2\,10^9 M_\odot \left(\frac{f}{10^{-2}}\right)^{-1} \left(\frac{\sigma}{300\,\mathrm{km\,s^{-1}}}\right)^5 . \qquad (4)$$

Such values, consistent with those adopted in our computations, for f in the range $3-5\,10^{-2}$ gratifyingly agree with the masses of dark objects detected at the center of many galaxies. Also the trend accords with the correlation first pointed out by Ferrarese & Merritt (2001) and Gebhardt et al. (2001). Note that on using the second approximation to $M_\bullet$ discussed below eq. (3), our correlation is somewhat flatter than $M_\bullet \propto \sigma^5$; it has the slope 4.3 around $\sigma \approx 300$ km s^{-1}, and the prefactor $3\,10^9\,M_\odot$.

Next we discuss how our limiting value $\Delta E/E \approx 1$ arises in galaxies from accretion regulated by the AGN itself (see also Silk & Rees 1998). On the one hand, sustaining $\Delta E/E$ to about 1 requires sufficient cold gas made available for inflow. The requirement is met by gravitational torques exerted in the host by companion galaxies within small groups during an encounter or a flyby over times of order t_d, see Cavaliere & Vittorini (2002). Such interactions destabilize fractional gas masses of order 10^{-2}, while the values needed to satisfy eq. (4) in the host galaxies are only of order $\sigma^2/f\,\eta c^2 \approx 2\,10^{-3}\,(\sigma/300\,\mathrm{km\,s^{-1}})^2$.

On the other hand, $\Delta E/E$ will be bounded when the accretion on to the central BH can be limited on a time scale t_d by the AGN feedback. If so, the AGN will fade out; declining luminosities are included in our self-similar blastwave family under the form $L \propto t^{5(2-\omega)/\omega}$ if the initial density gradient follows a steeper law $n \propto r^{-\omega}$ with $\omega \geq 2$. Increasing ω up to 2.5 corresponds to $L(t)$ going from constant to a spike; up to $\omega \approx 2.4$ the non-linear behavior of $L_X - T$ around $\Delta m/m \approx 1/2$ is generic.

But when ω approaches 2.5 the effective time scales become quite shorter than t_d, while at a given $\Delta E/E$ the blastwave is found to cause larger values of $\Delta m/m$. This behavior is indicative of runaway conditions prevailing when a galaxy happens to grow a large BH in short times. Then most galactic gas is blown away far outward R_v, so the star formation activity is suppressed at $z \lesssim 2$ (see also Granato et al. 2001). Such may have been the case for some of the recently discovered EROs (Cimatti et al. 2002; Alexander et al. 2002).

To conclude, we put in context and summarize our findings. The DM clusters hierarchically under its own weak gravity, and behaves in the scale-invariant manner reflected in the nearly universal shape of the potential wells $\Delta\phi$. The baryons, however, behave quite differently due to the energy fed back as they condense under *strong* gravity. The latter cooperates with the other fundamental interactions in the star cores that re-explode as type II SNe; but it acts in its purest form in the galactic relativistic dips accreting gas that energizes the AGNs.

In the latter case, the *shapes* of the $L_X - T$ and of the $M_\bullet - \sigma$ correlations are strongly affected and *linked*. This is because in moving from clusters to groups the energy injected by an AGN over a time t_d grows relative to the unperturbed one, and dominates over SNe. But on entering the galactic range values $\Delta E/E \approx 1$ arise and begin to constrain the accretion. These correlations provide two linked but observationally independent *probes* of the hidden parameter f; the current X-ray and optical data indicate values around $f \approx 5\,10^{-2}$.

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