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MERGING RATES INSIDE LARGE-SCALE STRUCTURES

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ABSTRACT

We use data from cosmological N-body simulations to determine amplitude and time evolution of the galaxy number density and relative velocity inside large-scale filamentary structures. We use the above determinations in analytical formulas to evaluate the galaxy merging rate in such environments where most of the galaxies are found. The resulting rate does not decrease with time (as it would in a Hubble expanding field) and is of order of $\sim$5–10 Gyr$^{-1} \gg H_0^{-1}$ so that the evolution of galaxy statistics can be significantly affected by binary aggregations.

Subject headings: galaxies: clustering — galaxies: interactions — methods: numerical

1. INTRODUCTION

The evolution of cosmic structures such as galaxies, groups, and clusters has been mainly treated in terms of direct collapse of overdense regions. Correspondingly, the statistics of the collapsed objects is treated in terms of a Gaussian perturbation field with random phases and power spectrum $\langle |\delta_k|^2 \rangle = P(k)$. The properties of the field are known from the linear theory of perturbations (Peebles 1980).

This approach seems successful in describing the statistics of structures at a given mass as they arise from the density field. However, their later evolution may considerably change their mass distribution. In particular, merging has been thought to be important for the evolution of systems embedded into a larger ambient structure (Toomre 1977). Numerical simulations by Cavaliere et al. (1986), Efstathiou et al. (1988b), and West, Oemler, & Dekel (1988) discuss aggregation of subclusters in clusters, while Carnevali, Cavaliere, & Santangelo (1981), Ishitzawa et al. (1983), Merrit (1983), Richstone & Malumuth (1983), Barnes (1989), and Mamon (1990) have studied merging of galaxies in rich environments. The complex dependence on the environment of the evolution under binary aggregation has been analytically studied by Cavaliere, Colafrancesco, & Menci (1992, hereafter CCM) using the kinetic Smoluchowski equation, which describes the time evolution of the mass distribution following merging events that take place with an aggregation rate $\tau^{-1} = \rho\Sigma V$ (see below).

It turns out that the mass distribution is considerably modified by the repeated merging only in those environments where the galaxy number density ρ does not rapidly decrease with time, as is expected because aggregation is a binary process. The density and the relative velocity are easily computable in well-defined virialized environments, as is the case for galaxies in groups and in clusters (CCM). In fact, in those cases well-established models exist for the evolution and distribution of the density and velocity of the aggregating bodies. Given those inputs, the dynamics is defined by the Smoluchowski equation, and several results have been worked out (CCM; Cavaliere & Menci 1993). In environments with small velocity dispersion the merging evolution can end up in a phase transition leading to the formation of a large merger (cD-like galaxy) surrounded by satellites, or to the erasure of substructures. In environments with large velocity dispersion, aggregation may considerably flatten the mass distribution and stimulate star formation, thus providing a possible explanation of the Butcher-Oemler effect (see Butcher & Oemler 1984).

Recently, however, merging has been considered responsible for the evolution of the galaxy distribution even in the field. This could help us to understand how a locally flat luminosity function $N(L)$ (Efstathiou, Ellis, & Peterson 1988a; Binggeli, Sandage, & Tamman 1988) may arise from an originally steeper (Press & Schechter 1974) mass distribution still producing the faint B counts (see discussions by Broadhurst, Ellis, & Shanks 1988; Lacey & Silk 1991), the latter being also constrained by a modest $\langle z \rangle \approx 0.3$–$0.4$ (Colless et al. 1990; Cowie, Songalia, & Hu 1991). Several authors (Rocca-Volmerange & Guiderdoni 1990; Guiderdoni & Rocca-Volmerange 1991; White & Frenk 1991) suggest that these problems could be better understood if the mass distribution is considerably reshaped by merging.

However, in the Hubble expanding field the density drops like t^{-2} in a critical Friedmann universe, while merging requires a protected environment to be effective. It has then been suggested that such environments could be provided by large-scale cellular structures, like filaments and sheets, that appear both in the deep redshift surveys (Geller & Huchra 1988; Ramella, Geller, & Huchra 1989) and in the pencil beam surveys (Broadhurst et al. 1990), as well as in large-scale N-body simulations (see, e.g., White et al. 1987).

In this paper we use the data from large-scale N-body simulations of the standard cold dark matter (CDM) universe to study the large-scale structures (filaments) as possible environments for the merging activity. We do not have enough resolution to both include the filaments in the simulation volume and resolve the galaxy-galaxy interactions (which involve the galaxy internal degrees of freedom). However, the dynamics due to binary aggregation is fully determined by the Smoluchowski kinetic equation once the merging rate has been specified. This depends on the amplitude and the time evolution of the galaxy number densities and on the relative velocities and scaling with mass of their cross section for merging. Thus, we measure these quantities from the simulation data

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inside large-scale structures (where no analytical models exist for their determination) and evaluate the evolution of the merging rates in such environments. Once the latter is measured, the dynamic evolution is determined.

2. DEPENDENCE OF MERGING RATES ON THE ENVIRONMENT

The evolution of galaxy mass distribution due to aggregation processes can be described by the Smoluchowski equation, which has the form

$$\frac{\partial N(M)}{\partial t} = \frac{1}{2}\,\overline{NN\rho\langle\Sigma V\rangle} - \overline{NN\rho\langle EV\rangle}\,, \tag{1}$$

where the bar denotes integration over mass, ρ is the density of objects in a given environment, Σ is the cross section for merging, and $\langle\,\rangle$ indicates the average over the relative velocities V. In addition to the mass conservation in two-body interactions, physics enters in the equation *only* through the aggregation rate

$$\tau^{-1} = \rho\langle\Sigma V\rangle\,, \tag{2}$$

The cross section Σ for merging of two members of masses M and M' and radii r and r' is (Saslaw 1985)

$$\Sigma \approx \pi(r + r')^2[1 + 2G(M + M')/(r + r')V^2]\,. \tag{3}$$

For large values of V the cross section is purely geometrical, while when V is of the order of $(GM/r)^{1/2}$ (the velocity dispersion of the star in the galaxies) the focusing effect of gravity is dominant.

Following CCM, using the scaling $r \propto (M/\rho)^{1/3}$ we express τ^{-1} as a function of ρ, V, and M only, to obtain

$$\tau^{-1} \propto \rho V M^{-1/3} \quad \text{and} \quad \tau^{-1} \propto \frac{\rho}{V} M^{1/3}\,, \tag{4}$$

in the limit of large and small relative velocity V, respectively. They correspond to the limit of geometric cross section and of focused cross section.

We also parameterize the dependence of the rate on the two variables t and M in the following way

$$\tau^{-1} \propto t^f M^{\lambda-1}\,. \tag{5}$$

The parameters f and λ depend strongly on the environment. In particular, f results from the evolution of the density $\rho \propto t^u$ and of the relative velocity $V \propto t^s$ of galaxies in their environment, while λ is $\frac{1}{3}$ for small V/v (v is the galaxy internal velocity dispersion) and $-\frac{1}{3}$ for large V/v.

Once the two parameters have been evaluated, the dynamics of the system according to equation (1), i.e., the evolution of the mass distribution due to aggregation, is fully determined.

Such computations show that the evolution of the system is critically sensitive to the above parameters. If $\lambda > 1$, a phase transition can take place, and the system breaks into a bimodal distribution constituted by a merger surrounded by smaller satellites. This situation may be related to the formation of cD-like galaxies or to the erasure of substructure in a cluster (see CCM). If $\lambda < 1$ holds, the mass distribution tends to flatten, reaching an asymptotic shape $m^{-1.2}\exp(-m)$, where $m = M/M_*$ and $M_* \propto t^{(1+f)/(1-\lambda)}$ is the characteristic mass for aggregation. The latter situation may apply to galaxies in clusters and be responsible for the Butcher-Oemler effect as well as contribute to the local shape of the galaxy luminosity function.

Also the parameter f has a critical value. This is even more crucial in determining the effectiveness of systems under merging interactions. In fact, when f is below the critical value $f = -1$ the mass distribution is not changed at all by merging (CCM). This simply reflects the fact that if the density of objects drops too rapidly they are not able to meet each other and merging is not effective. Note that if the environment ambient is constituted by the freely expanding field, the density drops like t^{-2} so that, unless we have a very particular velocity field, $f < -1$ and merging is not effective. Thus environments protected from Hubble expansion are required. These may be constituted by virialized structures, like groups and clusters of galaxies, or by the less well defined (but extremely populous) large-scale structures.

The above parameters are relatively easy to compute when the environment is a well-defined, virialized structure, as is the case for groups and clusters of galaxies (see CCM), but they have not been explored for environments constituted by filamentary large-scale structures, wherein most of the galaxies are found.

3. *N*-BODY INTEGRATIONS

In order to measure the above parameters in large-scale cosmic structures we ran a set of N-body integrations for a CDM model. The simulations we use are with $\Omega_0 = 1$ and $H_0 = 50$ km s^{-1} Mpc^{-1}. The particle positions are integrated foward in time with a P^3M code, and we take $N_p = 64^3$ particles and $N_g = 160^3$ grid points, with periodic boundary conditions. The initial coordinate displacements are obtained using the Zel'dovich algorithm, the spectrum δ_k has a Gaussian distribution with random phases and $\langle|\delta_k|^2\rangle = P(k)$. Here $P(k)$ is the power spectrum for CDM. The comoving box size is $L = 360$ Mpc, and the softening parameter is 75 kpc $= L/480$. The expansion factor $a(t)$ scales as $t^{2/3}$, and at the initial time t_i we set $a(t_i) = 1$. We ran five integrations each with a different random realization of δ_k. At $a(t_i) = 3$ the power spectrum amplitude is such that the rms mass fluctuation, filtered on a 16 Mpc scale, is $\sigma_m = 1/b$, where $b = 2.5[3/a(t)]$ is the bias factor. These simulations are similar to the ones of Frenk et al. (1990).

To each particle, at $a(t_i)$, we associate a "peak number" n_i calculated from the initial density field; the statistical approach is the same as in White et al. (1987). Here n_i is the number of "galaxy" peaks with height above $v\sigma_s$, for the field $\delta(x)$ Gaussian-smoothed with $r_s = 1.1$ Mpc. The field is subject to the constraint that it takes the value $v_b\sigma_b$ when smoothed on a scale $r_b > r_s$ (see, e.g., White et al. for more details).

We choose $v = 1.5$, and the particle two-point correlation function, weighted according to n_i, matches in slope and amplitude that of galaxies when $a = 4.5$. Thus the bias factor at the present epoch is $b = 1.67$.

4. MEASURING THE PARAMETERS FOR MERGING

To estimate the key parameter f in filaments we need first to select the galaxies that are in such structures. We used a standard FOF (friend-of-friend) algorithm (Einasto et al. 1984) on the simulation particles to select the large-scale structures. Following the prescription by Einasto et al. and by Dekel & West (1985) we used a linking parameter $l/\bar{l} = 0.7$ ($\bar{l}$ is the average particle separation in the simulation volume). The selected structures are filaments of the kind shown in Figure 1. The number density and relative velocity of the galaxies in the filaments are computed at each cosmic time.

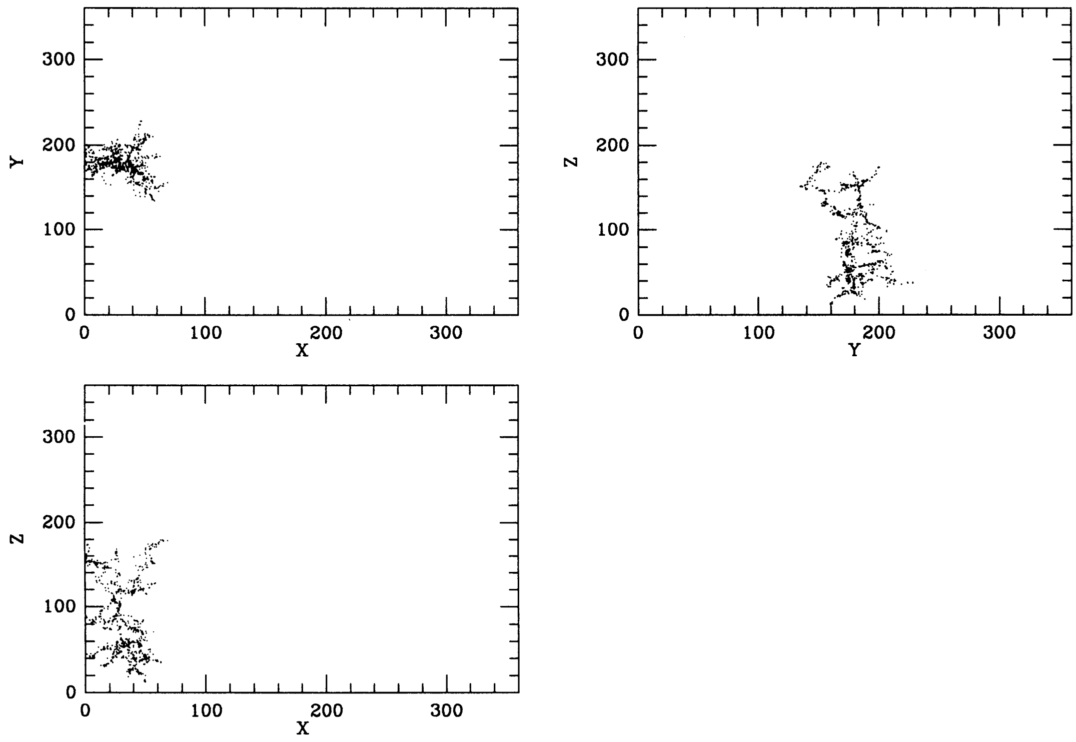

FIG. 1.—One of the largest filaments selected from the simulations projected onto the three orthogonal planes (units are in Mpc). The structures are generally elongated with typical dimensions ~ 100 Mpc.

The number of galaxies $n^i_{\rm gal}$ associated to the ith simulation particle is proportional to the peak number n_i (defined above) associated with that particle. The proportionality constant α is fixed by the requirement $\Sigma n^i_{\rm gal} = \alpha \Sigma n_i = N_{\rm tot}$, where $N_{\rm tot} =$ 43,000 is the total number of galaxies in the simulation volume corresponding to an average number density equated to that at the median distance in a magnitude-limited catalog (White et al. 1987). With our prescription $n^i_{\rm gal} < 1$ actually represents the probability that a galaxy is assigned to the ith simulation particle. Thus, we assign a galaxy to the ith particle each time $n^i_{\rm gal}$ is larger than a random number extracted in the interval [0, 1].

The number of bright galaxies in a 360 Mpc^3 volume is likely to be larger than our $N_{\rm tot}$, so our conservative assumption about the galaxy number will lead to an underestimate of the true merging rate τ^{-1}.

To write the number density of galaxies in a given filament, let us indicate by $\{J\}$ the set of simulation particles that are found to be in the Jth filament[2]

$$\rho_J = \frac{\text{no. galaxies in filaments}}{\mathscr{V}_J} = \frac{N_J}{\mathscr{V}_J} \frac{\sum_{i \in \{J\}} n^i_{\rm gal}}{N_J} , \tag{6}$$

where N_J is the total number of simulation particles in the volume $\mathscr{V}_J$ of the Jth filament and

$$\overline{n_J} \equiv \frac{\sum_{i \in \{J\}} n^i_{\rm gal}}{N_J} \tag{7}$$

is the average number of galaxies associated with a simulation particle in the Jth filament. The ratio $N_J/\mathscr{V}_J$ is the inverse average particle separation in the filament $1/\bar{l}_J$. Thus, the numerical galaxy density in the Jth filament is

$$\rho_J = \overline{n_J}/\bar{l}_J , \tag{8}$$

where the quantities on the right-hand side are both easily measurable.

In a given filament we also measure the relative velocity for each pair of galaxies, i.e., the component of the velocity in the direction connecting the two galaxies; $v_{ij} = \boldsymbol{r}_{ij} \cdot (\boldsymbol{v}_i - \boldsymbol{v}_j)/r_{ij}$. For each filament we compute the average relative velocity $V_J = \langle v_{ij} \rangle$.

With V_J and ρ_J known, the merging rate in a given filament is computed from equation (2) using the full expression (3) for the cross section. The galactic radii are assumed to all have the same (conservative) value $r_i = 30$ kpc, and the mass of a simulation particle is $M = 1.3 \times 10^{13}\, M_\odot$.

5. RESULTS

Here we present the results relevant to estimating the merging parameters, from a simulation of a CDM universe.

In Figure 2 the mass function of the filaments selected by the FOF algorithm is illustrated at five different times corresponding to different expansion factors. As time increases, larger and larger structures arise from the field, and the number of small filaments ($M \sim 5 \times 10^{15}\, M_\odot$) increases.

The ratio of the internal galaxy number density ρ_J in a filament to the total (in the whole simulation volume) number density $\rho_{\rm tot} \propto t^{-2}$ is plotted as a function of time in Figure 3. We note that the density contrast follows a definite behavior

[2] In the following we will indicate with capital letters the indices associated with the filaments.

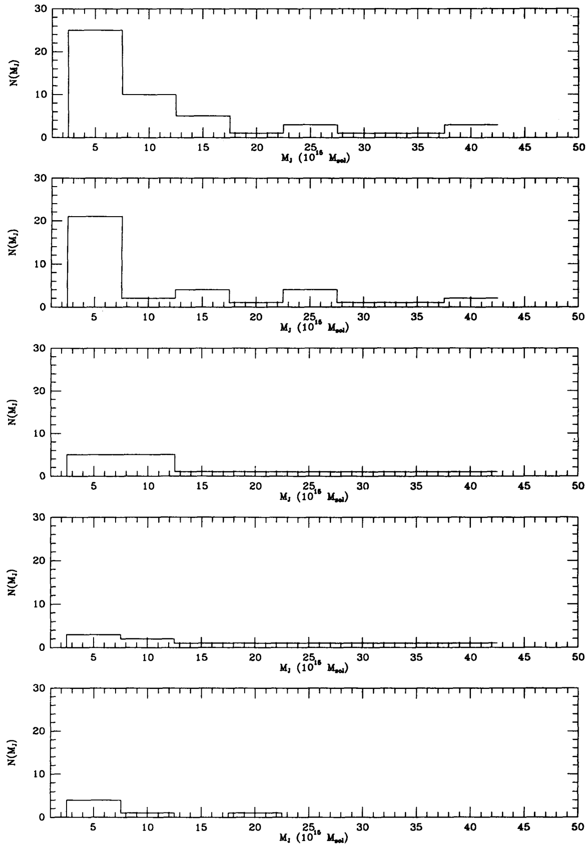

FIG. 2.—Mass function of the selected filaments at times $t/t_0 = 0.35$, 0.53, 0.77, 1, 2.1 (from bottom to top), t_0 being the present epoch. The corresponding redshifts are $z = 1, 0.5, 0.18, 0, -0.4$.

which is the same for all selected filaments. The density contrast averaged over all filaments selected at a given time is plotted as a function of time in Figure 4, where it is compared to two limiting cases: constant internal density of the filaments and internal density which follows exactly the Hubble flow $\rho_J \propto \rho_{tot} \propto t^{-2}$ (in a critical universe). The logarithmic scale

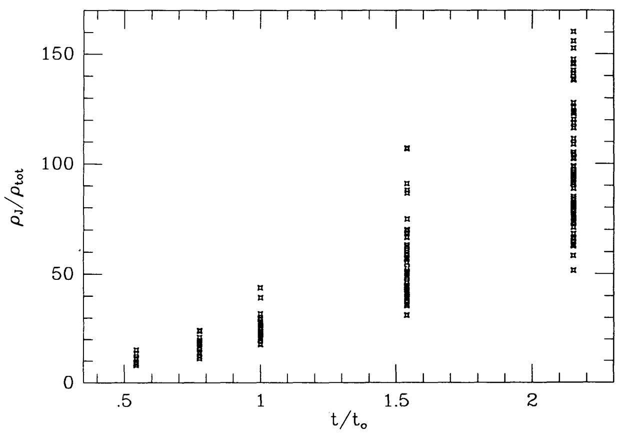

FIG. 3.—Galaxy number density inside filaments ρ_J normalized to the total galaxy number density ρ_{tot} in the simulation volume, is plotted as a function of the cosmic time t (divided by the present time t_0). Each spot correspond to a filament selected from the simulation.

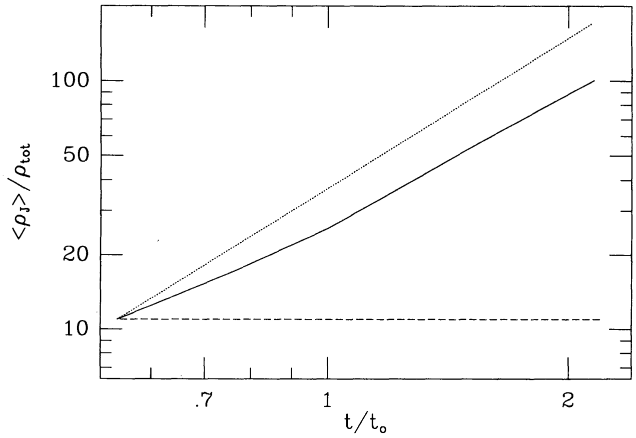

FIG. 4.—Ratio $\langle\rho_J\rangle/\rho_{tot}$ (*solid line*) as a function of t. The average is over all the set of filaments $\{J\}$ selected at a given time. The behavior is compared with two limiting cases: a constant internal density ρ_J (*dotted line*) corresponding, in a critical universe, to $\langle\rho_J\rangle/\rho_{tot} \sim t^2$ and an internal density following the Hubble expansion $\langle\rho_J\rangle \propto \rho_{tot}$ (*dashed line*).

stresses that the internal density is far from following the Hubble flow. For times smaller than the present it follows an approximate power law $\rho_J \propto t^{-0.4}$, and at future times it tends to become constant with time.

But, for merging to be efficient, conditions of slow expansion must be satisfied not only on average, but also within a given environment. That is, permanence of a filament for times longer than the galaxy merging time τ is needed. To assess whether this is the case, we have partitioned each of the filaments found at the present time t_0 into subsets containing N_1, N_2, ... particles with $N_1 > N_2 > \ldots$ and $N_t = N_1 + N_2 + \ldots$ (see Frenk et al. 1988). The subsets are defined by the particles that, at a given time $t < t_0$, belonged to different filaments. We plot $\log(N_t/N_1)$ as a function of $\log(N_1/N_2)$ in Figure 5.

If $N_t \approx N_1$ almost all the particles found in the filament at t_0 come from a *unique* progenitor at t; i.e., the filament population is slightly affected by merging or accretion. The relative importance of these mechanisms is estimated by the ratio N_1/N_2. When this tends to 1 the filament population is built mainly by the merging of two subsets of comparable size, while for larger values accretion of small groups of galaxies onto a stable structure dominates over merging.

The plots in the figure show that, for all $t < t_0$, the filaments are stable structures that maintain their identity during their history while the scatter in N_1/N_2 is confined to larger values.

So the large-scale structures may in fact provide a protected environment where merging is efficient. To fully determine the evolution of the merging rate one needs to compute the average relative velocity. The result is plotted in Figure 6, where the average is again computed over all the structures selected at a given time t. From this plot we can extract further important information. The relative velocity, although decreasing with time, is considerably larger (at least for not too large cosmic times) than the internal galaxy velocity dispersion ($\sim$400 km s^{-1}), at least for times up to the present. From what we have shown in § 2, this means that the value of the λ parameter (defining the merging regime) is $-\frac{1}{3}$, so that the interactions in practice follow a geometrical cross section instead of the focused one (which is typical of low velocity dispersion environments like groups of galaxies).

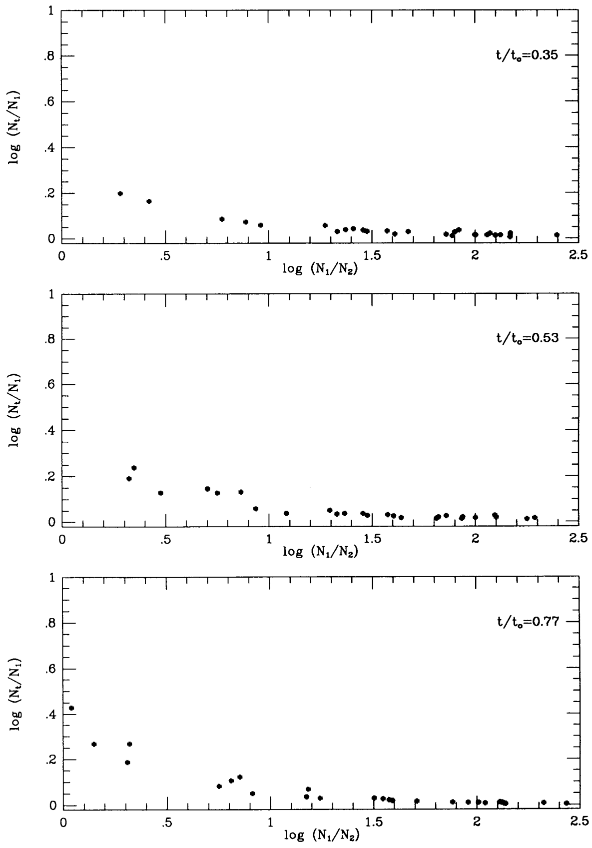

FIG. 5.—Ratio N_t/N_1 as a function of N_1/N_2. N_t is the number of particles in a given filament at the present time t_0, while N_1 and N_2 are the number of galaxies in the two largest progenitor structures at the time t. Each dot corresponds to a filament.

Thus, the merging rates can be obtained by combining the results on the evolution of the density of galaxies and of their relative velocity in the filaments, illustrated in Figures 4 and 6, using the first of the equations (4) appropriate for large V and corresponding to a geometrical cross section, with M given in § 4.

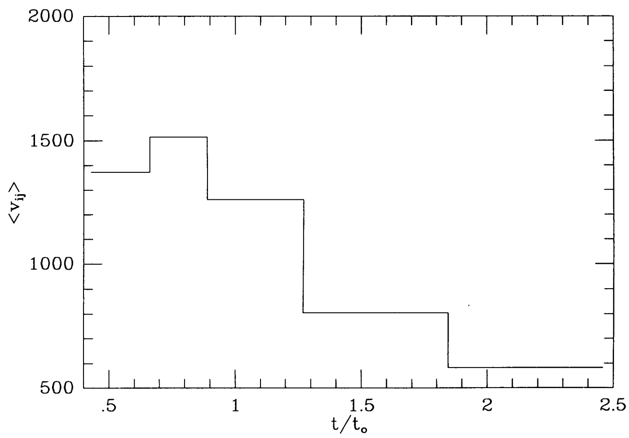

FIG. 6.—Histogram shows the time evolution of the average (over all selected filaments at time t) galaxy-galaxy relative velocity (component parallel to the connecting direction).

The time evolution and magnitude of the resulting merging rates are shown in Figure 7 for four different filament mass ranges. It is evident that the rate is never decreasing with time for $t \leq t_0$. For sufficiently large filaments the value of the parameter f discussed in § 2 (describing the evolution) is ≈ 0 (no evolution of the rate). In all cases the rate is $\gg H_0^{-1}$ so that the evolution of galaxy statistics can be significantly affected by binary aggregations.

6. CONCLUSIONS

Using data from CDM N-body simulations, we have shown that the large-scale structures modulating the field do contribute a favorable environment for merging. We have chosen the present epoch to correspond to a bias factor $b \simeq 1.5$ in order to span, with the present simulation, the largest possible range of times. Choosing an higher bias at the present epoch does not alter our results in a significant way.

We estimated, inside large-scale structures, the parameters which, in a kinetic theory of merging interactions (see § 2) completely define the dynamics, namely, the *amplitude* and the *time evolution* of the relative velocity of galaxies and of the density field. The merging regime, i.e., the kind of cross section which is effective for the interactions, depends on these quantities (see § 2).

The results indicate that the relative velocity decreases with time, with an amplitude which makes the geometrical part of the merging cross section (eq. [3]) dominant. The number density of galaxies found in the structures at each cosmic time t is decreasing with time ($\rho_J \sim t^{-0.4}$) for $t < t_0$ and stays roughly constant for $t \geq t_0$.

The average results reflect the actual history of the single filaments. In fact, we showed in § 5 that filaments tend to conserve their identity over periods larger than the galaxy merging time τ, this providing a stable and protected environment for merging.

With such conditions, merging is efficient and the merging rate is approximately constant with time. This has several cosmological implications.

Merging actually constitutes *in general* a second, relevant mode of galaxy evolution in the cosmological environments where most of the galaxies are found (bound structures and large-scale structures). Therefore, one may also expect the galaxy mass function to be substantially reshaped by merging subsequent to galaxy formation. Starting at $z \sim 1$–2 with a Press & Schechter–like mass function, the quantities that we estimate in the present paper—$V(t)$, $\rho(t)$, and $\Sigma(M)$—lead to a dynamical evolution (describable by the kinetic eq. [1]) which predicts a continuously flattening galaxy mass distribution $N(M)$ (Cavaliere & Menci 1993).

This form of strong density evolution has been phenomenologically considered by several authors (see Broadhurst et al. 1988; Guiderdoni & Rocca-Volmerange 1990) and has interesting consequences for understanding the observed large number of faint blue galaxies. The above observations are difficult to reconcile with an $\Omega = 1$ universe, the discrepancy not being completely explained by the models of spectroscopic evolution (see, e.g., Rocca-Volmerange & Guiderdoni 1990, and references therein; Cowie et al. 1991) proposed so far. As pointed out by the cited authors, an effective merging evolution, in addition to a luminosity evolution, could alleviate or solve the above problems.

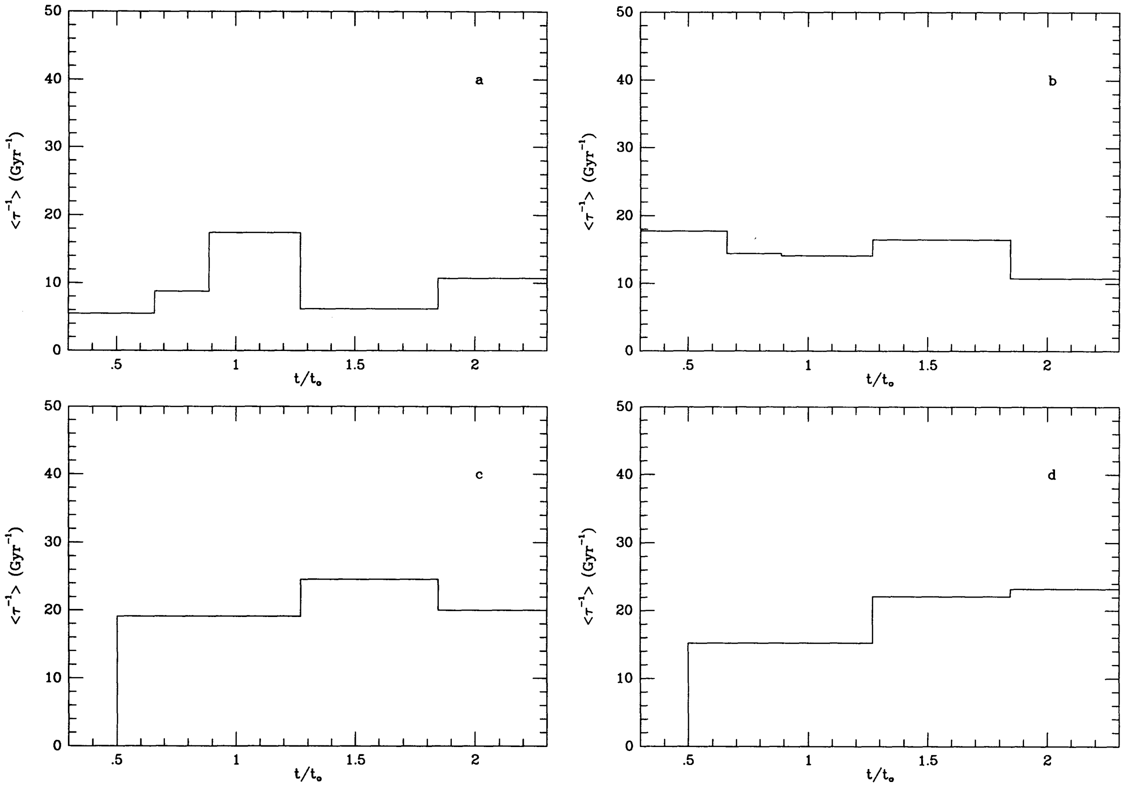

FIG. 7.—Average merging rate τ^{-1} as a function of time for four different filament mass ranges. Panels *a–d* correspond to $M_J < 5 \times 10^{15}\ M_{\odot}$, $M_J = (5 \times 10^{15}$ to $1.5 \times 10^{16})\ M_{\odot}$, $M_J = (1.5\text{–}2.5) \times 10^{16}\ M_{\odot}$, and $M_J > 2.5 \times 10^{16}\ M_{\odot}$, respectively.

On the other hand, recent large-scale data (Peacock & Nicholson 1991; Vogeley et al. 1992; Jing & Valdarnini 1993; Fisher et al. 1993) and the *COBE*-DMR measurements (Smoot et al. 1992) seem to require more power on large scales than the standard, biased, CDM model. Thus filaments and large-scale structures inevitably occur and are likely to be the sites of an even higher merger activity than the one modeled here.

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REFERENCES

Barnes, J. E. 1989, Nature, 338, 123
Binggeli, B., Sandage, A., & Tamman, G. 1988, ARA&A, 26, 508
Broadhurst, T. J., Ellis, R. S., Koo, D. C., & Szalay, A. S. 1990, Nature, 343, 726
Broadhurst, T. J., Ellis, R. S., & Shanks 1988, MNRAS, 235, 827
Butcher, H., & Oemler, A. 1984, ApJ, 285, L45
Carnevali, P., Cavaliere, A., & Santangelo, P. 1981, ApJ, 340, 47
Cavaliere, A., Colafrancesco, S., & Menci, N. 1992, ApJ, 392, 41 (CCM)
Cavaliere, A., & Menci, N. 1993, ApJ, 407, L9
Cavaliere, A., Santangelo, P., Tarquini, G., & Vittorio, N. 1986, ApJ, 305, 651
Colless, M., Ellis, R. S., Taylor, K., & Hook, R. N. 1990, MNRAS, 244, 490
Cowie, L. L., Songalia, A., & Hu, E. M. 1991, AA, 252, 435
Dekel, A., & West, M. J. 1985, ApJ, 288, 411
Efstathiou, G., Ellis, R. S., & Peterson, B. A. 1988a, MNRAS, 232, 431
Efstathious, G., Frenk, C., White, S. D. M., & Davis, M. 1988b, MNRAS, 235, 715
Einasto, J., Klypin, A. A., Saar, E., & Shandarin, S. F. 1984, MNRAS, 206, 529
Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1993, ApJ, 402, 42
Frenk, C., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Frenk, C., White, S. D. M., Efstathiou, G., & Davis, M. 1990, ApJ, 351, 10
Geller, M. J., & Huchra, J. H. 1988, in Large-Scale Motions in the Universe: A Vatican Study Week, ed. V. C. Rubin & G. V. Coyne (Princeton, NJ: Princeton Univ. Press), 3
Guiderdoni, B., & Rocca-Volmerange, B. 1991, MNRAS, 252, 435
Ishizawa, T., Matsumoto, R., Tajima, T., Kageyama, H., & Sakai, H. 1983, PASJ, 35, 61
Jing, J. P., & Valdarnini, R. 1993, ApJ, 406, 6
Lacey, C., & Silk, J. 1991, ApJ, 369, 79
Mamon, G. A. 1990, in IAU Symp. 124, Paired and Interacting Galaxies, ed. J. W. Sulentic, W. C. Keel, & C. M. Telesco (NASA CP-3098), 609
Merrit, D. 1983, ApJ, 264, 24
Peacock, J. A., & Nicholson, D. 1991, MNRAS, 253, 307
Peebles, P. J. E. 1980, The Large Scale Structure of the Universe (Princeton, NJ: Princeton Univ. Press)
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Ramella, M., Geller, M. J., & Huchra, J. P. 1989, ApJ, 344, 57
Richstone, D. O., & Malumuth, E. M. 1983, ApJ, 268, 30
Rocca-Volmerange, B., & Guiderdoni, B. 1990, MNRAS, 247, 166
Saslaw, W. C. 1985, Gravitational Physics of Stellar and Galactic Systems (Cambridge: Cambridge Univ. Press)
Smoot, G. F., et al. 1992, ApJ, 371, L1
Toomre, A. 1977, in The Evolution of Galaxies and Stellar Population, ed. B. M. Tinsley & R. B. Larson (Cambridge: Cambridge Univ. Press), 111
Vogeley, M. S., Park, C., Geller, M. J., & Huchra, J. P. 1992, ApJ, 391, L5
West, M. J., Oemler, A., & Dekel, A. 1988, ApJ, 327, 1
White, S. D. M., & Frenk, C. 1991, ApJ, 379, 52
White, S. D. M., Frenk, C., Efstathiou, G., & Davis, M. 1987, ApJ, 313, 505